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THE MATHEMATICS TEACHER

Volume XLIV Number 4

A Five Year Program of Preparation for Mathematics Teachers*

By MYRON F. ROSSKOPF
Syracuse University, Syracuse, New York

DURING the past ten years there has been more than usual interest in the problem of teacher training. The shortage of teachers brought on by total mobilization for our participation in the Second World War, the flight from teaching of men and women into industry and government service in the post-war economic expansion, and the increase in the number of children entering the schools of the nation served to keep this problem constantly before those responsible for teacher education. The gap between supply and demand of qualified teachers of science and mathematics is particularly important. All reports that have appeared since 1945 stress the problem of attracting a sufficient number of students into preparing themselves for teaching science and mathematics. In addition these reports give evidence of much dissatisfaction with the present program of teacher preparation. There is need for a new approach both to courses in the sciences and mathematics and to professional courses in education.

It is evident from a study of recent reports and articles that there is a strong movement toward a longer period of preparation for teachers. The President's Scientific Research Board indicates its point of view in the following words:

"The graduate offerings in science and mathematics are usually too highly specialized and narrowly defined to serve the needs of any individual who does not intend to become a research scientist.

"Some teacher training institutions should provide for at least one year of graduate work in which ample courses in science and mathematics are offered to meet the needs of the teacher instead of the research scientist.

"Such offerings will make it possible for many teachers who now are forced to do their graduate work in education only, to return to their teaching fields for a majority of their graduate study."[1]

Commenting on a part of the foregoing report, Professor Dodds of Purdue University writes, "It might even be suggested that this area of work is a true inter-departmental responsibility and that there are people available for university staffs who might hold joint appointments in both education and the subject matter department."[2] There is a belief that, "There should be more seminars in our teacher-training programs for the purpose

* Read at the Twenty-Eighth Annual Meeting of the National Council of Teachers of Mathematics at Chicago, Illinois, April 15, 1950.

[1] The President's Scientific Research Board, *Manpower for Research*, p. 107. Volume four of *Science and Public Policy*. Washington, D. C.: U. S. Government Printing Office, 1947.

[2] B. L. Dodds, "Cooperative Planning in Teacher Education," *School Science and Mathematics*, L (March, 1950), 226.

of integrating subject matter, and for the purpose of discussing the utilization of mathematics in other fields of knowledge."[3] The program that I am going to describe attempts to provide just those opportunities; that is, to provide an environment for student teachers that will facilitate integration of their knowledge of subject matter in their major area, psychology and professional education, and general education.

The all-university School of Education was established at Syracuse University in 1934.[4] From its inception a cardinal point of its philosophy has been furnishing future teachers with a unified and functional curriculum. An implication of this objective is that students have contact during their training period with a faculty that is sympathetic to this point of view. An immediate corollary is the appointment of personnel who will bridge the gap between a college and a school of education.

The personnel referred to in the foregoing paragraph were given the title "dual professor." The title emphasizes the sort of teacher training that a student obtains and underlines the philosophy of the School of Education. In each of the academic areas and in each of the special areas at least one dual professor has been added to the staff of the School of Education and the College of Liberal Arts or School of Fine Arts or any other college that is taking part in the training of teachers. The dual professors divide their time about equally between the two colleges. For example, the dual professor in mathematics teaches half his academic load in the mathematics department of the College of Liberal Arts and half his academic load in the School of Education. To talk about the academic load of a dual professor is to speak of the least important phase of his responsibilities. He is the cement that binds his department to the School of Education; thus, he establishes liaison between subject matter and teaching in a secondary school. In addition the dual professor supervises practice teaching and acts as adviser of juniors, seniors, and graduate students in his major area.

What of the students in the program? Does the School of Education welcome with open arms anyone who thinks he might like to teach in our public schools? No, the applicants are subjected to a well-organized selection program.[5] Applicants who are accepted become dual enrollees; that is, they retain enrollment in their first college choice and accept dual enrollment in the School of Education. During their freshman and sophomore years, the only contact that the faculty of the School of Education has with its probable future students is in the capacity of advisers or in a one-credit course, *Education Orientation*. Normally a student makes application for dual enrollment in the School of Education during the second semester of the sophomore year.

The Selection Committee is composed of three members of the School of Education faculty and a fourth member, the dual professor for the area in which the applicant intends to major. The folder of information that is considered when an applicant's admission is decided upon consists of two individual interview reports, one with the dual professor and one with the chairman of the Selection Committee, several independent interview reports on the applicant's group interview, and results on the following tests:—American Council on Education Test of Aca-

[3] C. V. Newsom, "Mathematics and Modern Educational Trends," THE MATHEMATICS TEACHER, XLII (November, 1949), 343.

[4] The Curriculum Committee of the School of Education, Syracuse University, *A Functional Program of Teacher Education as Developed at Syracuse University*. Washington, D. C.: American Council on Education, 1940. This monograph gives a complete report of the total education program, the several chapters treating in detail its phases.

[5] A detailed account of the selection program, together with an analysis of three cases, is given in a recent paper by Verna White, "Selection of Prospective Teachers at Syracuse University," *The Journal of Teacher Education*, I (March, 1950), 24–31.

demic Aptitude, Cooperative General Culture Tests, Cooperative Reading Test, Minnesota Multiphasic Personality Inventory, and the Strong Vocational Interest Inventory (for men) or the Kuder Preference Record (for women). An applicant is accepted; or he is rejected and guided through counselling into other outlets for his abilities and interests; or he is accepted on "release," pending the removal of weaknesses that his folder of information reveals. During the past two years, approximately 58 percent of those applying for admission to dual enrollment were accepted.

As was pointed out at the beginning of this paper, it has long been believed by forward-looking educators that a longer training period is necessary for teachers. Recently an important committee arrived at this statement, "Colleges and certification authorities should work toward a five-year program for the preparation of high school teachers. . . . There would then be more time for obtaining the broad liberal education which promotes growth of the student as a person and makes him a better teacher."[6] Under the urging of educational leaders the Board of Regents of the University of the State of New York adopted new requirements for certification in 1943. Basically these new requirements are summarized in the statement that five years of training now are necessary for certification to teach in the public schools of the state. Because of the shortage of qualified teachers during the next several years, enforcing the new requirements was postponed until 1951; then only students preparing to teach in the academic areas will come under the five-year requirement. Meanwhile the School of Education at Syracuse University moved ahead with plans for a training program that would not only satisfy the requirements of the law but would carry out the best thinking of educators. Committees were set up to study the present program, to search out its strengths and weaknesses, and to work out a philosophy for the training of the third, fourth, and fifth years of a prospective teacher.

The certification law states that thirty credit hours of advanced work beyond the bachelor's degree is required of all teachers of the academic subjects; the first application of the law will be made in 1951. As the faculty of the School of Education investigated this problem questions arose such as, "Could the present master's program be used to satisfy the new requirements? Can a three-year sequence of integrated experiences be developed that will better fit a teacher for his role in the classroom?" After examination of these questions and related problems, the faculty voted to re-plan the whole teacher preparation program in terms of a three-year sequence. Following is a short description of the courses in professional education, together with an account of the experiences in which all students participate.

Introduction to Human Development, Adjustment and Learning, 3 semester hours. This is the first course in the education sequence; it is taught during the first semester of the junior year. Through class discussion, work in Syracuse youth agencies, reading, and motion pictures, students obtain an understanding of the psychology of human adjustment and development.

Human Development, Adjustment and Learning, 4 semester hours. This course, taught during the second semester of the junior year is a continuation of the foregoing course. The major emphasis is on contemporary theories of the psychology of learning, In place of work in youth agencies students observe in Syracuse public schools. Each week a two-hour seminar is held by the dual professor for students majoring in his area. At this

[6] Report No. 4. The AAAS Cooperative Committee on Science Teaching, "The Preparation of High School Science and Mathematics Teachers," *School Science and Mathematics*, XLVI (February, 1946), 113.

time attention is directed to the application of the psychology of learning in the major area; observations are discussed and methodology of observed teachers is analyzed.

Creating the Learning Environment in the Classroom, 4 semester hours. During the first semester of his senior year, a student spends almost all of his time with his dual professor; the exceptions are seminars and laboratories in the two phases, audio-visual aids and equipment, and business machines and equipment. Emphasis is placed on methods of classroom teaching, evaluation in the major area, and careful planning for classroom work. Each student teaches one class per day for a period of six weeks in one of the Syracuse public schools.

The Exceptional Child, 3 semester hours. At this time, in the second semester of the senior year and after a practice teaching experience, it is believed the students are ready for an intensive study of the child as an individual. By means of class discussions, readings, class demonstrations, films, and field trips all the students have a chance to become familiar with characteristics of the normal as well as the exceptional child. Eyesight, hearing, and speech difficulties; rheumatic fever and cerebral palsy damages; heredity and environment factors; all these are considered in the course. The primary objective is not to make prospective teachers experts in these special fields but to make them sensitive to the problems that these children present in a classroom so that special considerations or special care can be recommended for them.

The Teacher as Educational Leader, 12 semester hours. Because extramural teaching requires that a student be away from the campus for six weeks, the first semester of the graduate year consists solely of courses in education. The faculty believes that it is easier for the School of Education to adapt its program to these special needs of its students than several liberal arts college departments. Although this block of work is considered an integrated whole, for purposes of instruction it is divided into five parts; these units are described in the following paragraphs.

The American School, Its Evaluation and Curriculum and *The American School: Its Organization and Administration* account for one-half the number of credits in the foregoing block. The titles indicate in an adequate fashion the content considered in these two units.

Interpretation of Educational Measurement, 2 semester hours. Emphasized in this unit is the practical knowledge of statistics that a teacher can use to analyze his classroom tests and to participate intelligently in a program of evaluation in a school.

Research in Teaching, 2 semester hours, and *Seminar in Teaching*, 2 semester hours, are conducted by the dual professor. Emphasized in the former are the research methods in the major field and reading of professional journals. Instruction as well as practice is given in organizing material for publication. During this semester the student, working closely with his dual professor, must prepare himself for living and teaching for a period of six weeks in a small community. Each student is responsible for teaching three classes per day, supervising study halls, attending faculty meetings, and helping the cooperating teacher in any way that might offer itself. After the students return to the campus from their extra-mural teaching there is planned for them a unique experience. In order that they may be familiar with the details incident to the taking of a field trip, they plan and take trips to such places as a bank, a television broadcasting station, a technical high school, the statistical department of a manufacturing plant, or a broker's office. It is believed that participation in these enrichment experiences will encourage them to use comparable community resources when they go out to positions as teachers.

Since most of our students come from urban communities and since most of

them will begin their teaching in a small community, a three semester hour specially organized course in *Rural Sociology* is also taken during the first semester of the graduate year.

In the second semester there is a two semester hour course in *Philosophy and Ethics of the Teacher* and a three semester hour course with the self-explanatory title of *Curriculum Workshop*. However, students will work in this curriculum workshop in both heterogeneous and homogeneous groups. Thus, they will have a chance to work with people in other subject matter fields, as on a school faculty, in studying curriculum problems, as well as working cooperatively with specialists in their own areas.

In addition to meeting all course requirements for the two degrees, bachelors and master's, a student must undertake a research project. The problem chosen is comparable to one that a classroom teacher might undertake. The research project may be an individual problem or, because of its nature, a cooperative attack by several students may be made. Examples of research projects that present students in the program have already begun to work on are a unit of work for general mathematics organized like a comic book, a test on critical thinking, and selection and organization of material for a consumer mathematics course in the senior high school.

A student majoring in mathematics is required to earn eighteen semester hours beyond the calculus to satisfy the requirements for a bachelor's degree. Courses such as *Advanced College Algebra, Infinite Series, College Plane Geometry, College Solid Geometry, Projective Geometry, Statistics, Introduction to Modern Mathematics, History of Mathematics, Fundamentals of Analysis,* and the like are included. Programs vary from student to student depending upon his background, abilities, and needs. The department has set nine semester hours as satisfying the mathematics requirement for the master's degree in education. Hence a student earns a total of twenty-seven semester hours in mathematics beyond the calculus by the time he finishes the five year program. Although a minor teaching subject is not required, each student is strongly urged to establish a minor in the sciences. There is ample room in each student's program for elective courses, either of a general education nature or in related areas of education like audio-visual aids and guidance.

At the present time students in the five year program are completing their fifth year. Even though it is too early to assess the effectiveness of the curriculum that has been designed for them, the evidence that has been gathered to date indicate that the faculty made a wise decision when it voted to plan an integrated program. In class discussions, readings, and practice teaching students show steady growth toward a mature understanding of boys and girls. Their work does not seem to them a series of disconnected courses. The whole program is an experience that forces them to study the problems of a teacher and to think about the implications for teaching that exist in a learning situation.

May Expirations

If the May issue of THE MATHEMATICS TEACHER is the last you are to receive until your membership is renewed, you have received notice of this. *Please renew your membership now.* If this is done before May 15 your name will *not* be dropped from the mailing list and you will be sure to receive the October issue. Thank you for your cooperation. HWC

Making "Pi" Meaningful

By HILLARD C. TRIPP
Appalachian High School, Boone, North Carolina

ONE OF the most difficult experiences that junior high school mathematics students encounter is the understanding of *pi*. The approximate value of 3.14 can be memorized in a short time, but what meaning does it have for the student after he has memorized it? As Professor Breslich points out:[1]

> Memorizing is not learning and drill alone is not conducive to learning.

The typical mathematics student has the idea that *pi* is a number, 3.14, which is used in working with circles. His concept of *pi* is quite vague. His experience with *pi* has been limited to using it in formulas for finding the circumference of a circle, the area of a circle, or the volume of a sphere. Unless he is an exceptional student, he does not readily comprehend the idea that *pi* is always the relationship between the diameter and the circumference of a circle.

Why is this memorization of *pi* not sufficient for the student's understanding if he can substitute 3.14 in a formula and get the correct answer? Again, as Professor Breslich states:[2]

> The pupil who in arithmetic has acquired the habit of relying upon rules of procedure that are presented in the textbooks or by the teacher, without training himself to gain real understanding of rules and processes, usually continues to memorize the rules in algebra and the demonstrations in geometry. He will fail because the processes of algebra are far more intricate than those of arithmetic. Unless teaching is such as to change these harmful methods cultivated by years of study of arithmetic he will not progress very far in high school mathematics.

Dr. Harold P. Fawcett raises the question concerning the guiding of the student in the exploration of this very important ratio.[3] Will the student be given time and opportunity to acquaint himself with this important ratio? Time should be given for the development of important concepts for nothing is gained when progress is too rapid for understanding.[4] The question then arises as to just how this very important concept can be made clear to the average junior high school mathematics student.

Human beings are endowed with five senses as well as with minds and the most effective approach to minds is by way of the senses.[5] It would seem, then, that if a situation could be set up by which the student could experience *pi* in more ways than just memorizing it, he would understand this ratio more clearly, and this concept would have a definite and lasting meaning to him. If he, himself, could actually measure the diameter and circumference of various sized circles, and then be guided in the arithmetical combination of these sets of numbers, his concept of *pi* would evolve as a natural part of his experiences.

The following guide sheet for the investigation of the relationship between the diameter (d) and the circumference (c) of any circle has proved to be quite beneficial for junior high school mathematics students in establishing the *pi* concept.

To have the maximum amount of understanding evolve from this experience, the student must have an understanding of approximate numbers and the meaning of

[1] Breslich, E. R., "Teaching Mathematics as a System of Understandings," THE MATHEMATICS TEACHER, Vol. XLII, No. 1, January, 1949, p. 61.

[2] *Ibid.* Breslich, p. 65.

[3] Fawcett, Harold P., "Mathematics in the Core Curriculum," THE MATHEMATICS TEACHER, Vol. XLII, No. 1, January, 1949, p. 10.

[4] Breslich, *op. cit.*, p. 63.

[5] Carnahan, Walter H., "Enrichment of Mathematics Teaching," THE MATHEMATICS TEACHER, Vol. LXII, No. 1, January, 1949, p 14

Guide Sheet For the Investigation of the Relationship Between the Diameter (d) and the Circumference (c) of Any Circle.

1. Measure the diameters and circumferences of ten or more circular objects to the nearest one quarter of an inch and tabulate these measurements in columns 3 and 4 below.

1	2	3	4	5	6	7	8
Circle No. or object	Instrument used to measure	C	D	$C+D$	$C-D$	$C\times D$	$C\div D$

2. Is there any one of the columns above that seems to result in approximately the same number? _____.
   If so which column? _____.
3. Take the average of the numbers in this column. Average _____.
4. What is the name of this constant (number)? _____.
5. Make a general statement concerning the relationship of the circumference and the diameter of any circle. State this also in a mathematical formula.

precision and accuracy in measurement.[6] It will also be necessary for the student to be familiar with the addition, subtraction, multiplication and division of mixed and decimal numbers, for his measurements will be taken in either one or the other of these numbers.

The day before the class begins their experience with *pi*, each student is asked to bring to class a tape measure or about three feet of string and a ruler. It is desirable to have at least one half as many circular objects of various sizes as there are students in the class. The class may be divided into groups of two and each member will have the opportunity of taking the measurements of the diameter and circumference of at least ten different sized circular objects.

Column No. 1 is filled out first in each case thus identifying each object measured. This is necessary in case a re-check is

[6] Butler and Wren, "The Teaching of Arithmetic," Chapter XI, *Teaching of Secondary Mathematics*, New York, McGraw-Hill, Series in Education, 1941, p. 249.

desired of any one of the measurements taken. In Column No. 2 the measuring instrument or instruments used may be the same for any one group of students throughout the investigation. However, this information is important, for each measuring device will have its inherent degree of error and this will be one of the reasons why the *pi* ratio will not be exact for each circular object. The measurements of the circumference and diameter are entered in Columns No. 3 and 4 respectively, preferably to the nearest quarter of an inch, if the measuring instruments allow for that degree of accuracy.

Some might question why other operations on the data are included in the guide sheet when only division produces a ratio and a result which is constant. If the division operation is the only one suggested on the guide sheet, the student may wonder what the result would be if these two special numbers were added, subtracted, or multiplied. If a nearly constant number results every time the circumference is divided by the diameter, the intellectual curiosity of the student may be aroused as to the possibility of the existence of other constants with other arithmetical combinations of these two related numbers. Let us not forget it is the student who is experiencing. When we can anticipate a question on the part of the student, let us subject him to experiences by which he can answer his own questions through a certain amount of guidance. Let him perform the addition, subtraction, and multiplication operations along with the division. This will give a contrast and make the circumference-diameter ratio more impressive in his experience.

Question No. 2 is inserted to direct the student's attention to the C/D ratio. In the third statement, the student gets the average of the ratios in Column No. 8 and arrives at a number that will be quite close to the value of *pi*, 3.14. The student may know what this constant is called from his past experience or the teacher may name it for him, giving some of the historical background of this ratio.[7]

The fifth and last question on the guide sheet gives the student the experience of summarizing and making a generalized statement concerning his experiences with the relationship of the circumference and diameter of a circle. He should be encouraged to make the statement in his own words and then transpose his generalized statement into a mathematical formula, $C/D=\pi$, or $C=\pi D$ by cross-multiplication.

The letter π should now have meaning to the mathematics student. When he sees it, his experiences have been such that he will recall the very important ratio of C/D. The concept of *pi* becomes a cornerstone in the structural foundation of the student's mathematical literacy.

[7] Bell, E. T., "The Age of Empiricism," Chapter 2, *The Development of Mathematics*, New York, McGraw-Hill Book Company, Inc., 1945, pp. 40, 46.

LETTER

A Suggestion

"I would like to offer one suggestion which would increase the value of the magazine—to me, at least.

"In order to enable those members who desire to file the various articles under subject matter to do so without destroying any articles, it would seem to be a good idea to arrange the format of the magazine so this could be done. Thus, if an article ends on an even-numbered page, no difficulty arises; the article can be cut out of the magazine and retained intact. If an article ends on an odd-numbered page, however, this cannot be done without losing something of either one or the other articles. This might be overcome by using the even-numbered page immediately following an article which concludes on an odd-numbered page for advertisements exclusively. In this fashion every article could be retained intact."

WALTER J. RISSLER
State Teachers College
Frostburg, Maryland

Linear Equations Without Determinants*

By C. C. MacDuffee
University of Wisconsin, Madison, Wisconsin

Determinants have become such a tradition in algebra that it is perhaps well to remind teachers of mathematics that the method of determinants is not in general the easiest way to solve systems of linear equations. In the special case of n consistent and independent equations in n unknowns, the method of determinants is very effective, but it is not well adapted to the treatment of special types of systems.

The primitive method of using elementary transformations, refined and systematized, is capable of producing the general solution of a system of any number of equations in any number of unknowns without preliminary separation into cases. All systems are treated by the same method. In the case mentioned at the beginning, this method is about as easy as the method of determinants, in most other cases it is easier. Moreover, it is simpler in theory and easier to teach.

Let us consider a typical problem, to find the general solution of

$$\begin{aligned} 2x+\ 3y-\ 5/2z+\ 5w&=2,\\ 3x+\ 5y-\ \ z+\ 2w&=3,\\ 7x+11y-\ \ 6z+12w&=7,\\ 3x+\ 4y-13/2z+13w&=3. \end{aligned}$$

There are three types of elementary transformation:

I. The interchange of two equations of the system.

II. The multiplication of each term of one equation by the same number $k\neq 0$.

III. The addition to each term of the ith equation of the system of l times the corresponding term of the jth equation of the system, $i\neq j$.

It is easily shown that an elementary transformation applied to a system of equations changes it into an equivalent system, two systems being equivalent if every solution of each is a solution of the other.

We shall define the *diagonal* of a system of equations as that sequence of coefficients which starts with the last coefficient in the last equation (not counting the constant term) and extends upward and to the left. Thus the diagonal in the system illustrated is

$$13,\ -6,\ 5,\ 2.$$

It will be shown that every consistent system in n unknowns can by means of elementary transformations be reduced to an equivalent system consisting of exactly n equations having the following properties, at which point the system is essentially solved:

1. Every coefficient above the diagonal is 0.
2. Every diagonal coefficient is 0 or 1.
3. Where the diagonal coefficient is 0, every other coefficient in that equation is 0.
4. Where the diagonal coefficient is 1, every coefficient below it is 0.

Let us return to our example. Since the second equation has the smallest coefficient of w, we may divide through by 2 and place this equation last:

$$\begin{aligned} 2x+\ \ 3y-\ 5/2z+\ 5w&=2,\\ 7x+\ 11y-\ \ 6z+12w&=7,\\ 3x+\ \ 4y-13/2z+13w&=3,\\ 3/2x+5/2y-\ 1/2z+\ \ w&=3/2. \end{aligned}$$

Clearly all the coefficients of w above the diagonal can be made 0 by adding -5 times the last equation to the first, -12 times the last equation to the second, etc. We obtain the equivalent system

* From a paper read before the Tenth Summer Meeting of the National Council of Mathematics Teachers, Madison, Wisconsin, August 22, 1950.

$$\begin{aligned} -11/2x-19/2y+\ 0z &= -11/2,\\ -\ 11x-\ 19y+\ 0z &= -11,\\ -33/2x-57/2y+\ 0z &= -33/2,\\ 3/2x+\ 5/2y-1/2z+w &= 3/2. \end{aligned}$$

The unusual feature of the above system is that, in making all coefficients of w above the diagonal equal to 0, we also made all coefficients of z in and above the diagonal equal to 0. The proper move now is to increase the number of equations to 5 by the introduction of a vanishing equation:

$$\begin{aligned} -11/2x-19/2y &= -11/2,\\ -\ 11x-\ 19y &= -11,\\ -33/2x-57/2y &= -33/2,\\ 0x+\ 0y+\ 0z &= 0,\\ 3/2x+\ 5/2y-1/2z+w &= 3/2. \end{aligned}$$

Now the term $-57/2y$ appears on the diagonal. To change this coefficient to 1, we multiply the third equation by $-2/57$, obtaining as the new third equation

$$11/19x+y=11/19.$$

We multiply this new third equation by 19/2 and add to the first, by 19 and add to the second. The system now takes the form

$$\begin{aligned} 0 &= 0,\\ 0x &= 0,\\ 11/19x+\ y &= 11/19,\\ 0x+\ 0y+\ 0z &= 0,\\ 3/2x+5/2y-1/2z+w &= 3/2. \end{aligned}$$

At this point the consistency or inconsistency of the system of equations will be evident. The constant terms (to the right of the =) have not been under our control. There are three equations whose left members vanish. If in one or more of these the right member did not vanish, the system would clearly be inconsistent. If these right members do vanish, as in our example, the system is consistent for a solution can be found.

We now drop or add vanishing equations until the system consists of exactly n equations. We have reached a system

$$\begin{aligned} 0x &= 0,\\ 11/19x+\ y &= 11/19,\\ 0x+\ 0y+\ 0z &= 0,\\ 3/2x+5/2y-1/2z+w &= 3/2 \end{aligned}$$

possessing the first three of our desired four properties. The fourth property may now be imposed. The first 1 on the diagonal above the last equation is the coefficient of y in the second equation. We add $-5/2$ times the second equation to the last (and in general continue this process from right to left) until every coefficient below a 1 in the diagonal is 0. Our system

$$\begin{aligned} 0x &= 0,\\ 11/19x+\ y &= 11/19,\\ 0x+0y+\ 0z &= 0,\\ 1/19x\ \ -1/2z+w &= 1/19 \end{aligned}$$

now has all four of the desired properties. It can be shown that this form is unique regardless of the steps taken to reach it.

Each unknown whose diagonal coefficient is 0 is a parameter, and each unknown whose diagonal coefficient is 1 is expressible in terms of these parameters. Thus in our example

$$\begin{aligned} x &= p,\\ y &= 11/19-11/19p,\\ z &= q,\\ w &= 1/19-1/19p+1/2q. \end{aligned}$$

This is a solution for every pair of values of p and q, and there are no others. This general solution may conveniently be written

$$\begin{aligned} (x, y, z, w) = (1, -11/19, 0, -1/19)p\\ +(0, 0, 1, 1/2)q\\ +(0, 11/19, 0, 1/19). \end{aligned}$$

It should be clear from this example how any system of linear equations can be solved.

Projects for Plane Geometry

By MARIE L. BAUER
Urbana High School, Urbana, Illinois

PROJECTS help the student to appreciate more fully the application of geometry to life situations, by the recognition of geometric form, the direct application of geometric principles and the use of geometric logic or methods in everyday reasoning.

Projects may take the form of posters, bulletin board displays, models, demonstrations, notebooks or booklets. In order to be interesting they should be well organized and illustrated. Illustrations may be cut from magazines or newspapers, or if he desires, the student may make his own sketches. These projects may be written up as themes, giving the illustration subordinate place, or the illustration may be made the center, using brief statements to point out the geometrical relationships.

The following topics will serve as a guide to the students. In some cases suggestions are given that may be followed, in others the ideas are left entirely to the student. A few of the topics overlap, however, they would probably be developed in an entirely different manner by different students. Two or three will be carried out to show what can be done. An attempt has been made to include a wide variety of subjects so that every student may choose a topic in his particular field of interest.

Since the purpose of the project is to give the student an opportunity to develop his own ideas on the topic he chooses, he is encouraged to find his own references, especially in modern magazines and newspapers. However, in order to aid those who find it difficult, a number of references are given, and with the exception of those where the title is self explanatory, a brief exception of those where the title is self explanatory, a brief explanation of what can be found in each is added.

1. Geometry on the Highways
   Use of curves and lines and the proper combination of the two for safety
   Cloverleaf highway intersections
   Geometry in highway signs, bridges, etc.
2. Geometry of Faces
   Basic types: a. Oval, b. Round, c. Square, d. Pear-shaped, f. Diamond-shaped, g. Oblong
   Optical illusions provided by the type of hair, dress and make-up
3. Geometry in the Home
   Geometric designs in rugs, linoleums, laces, quilts, furniture, pottery, jewelry, tablecloths, pillow tops, towels, clothes, textiles, blankets, wallpaper, mosaic floors and arrangement of furniture.
4. Geometry in Furniture
   Lampshades, desk tops, pictures, windows, chests, curtains, mirror, fireplace, chandelier, and chairs
   Arrangement of furniture for balance, symmetry or efficiency
5. Geometry in Drawing
   Blueprints, maps, and scale drawings
   This might also include cartoons using geometric figures
6. Geometry in Music
   Staff and symbols used in writing music
   Forms and designs of a number of musical instruments
7. Geometry in Fabrics
   Actual samples of materials, or designs drawn and painted are very effective
8. Geometry in Buttons
   Samples of actual buttons, pictures cut from magazines, or hand-drawn designs may be used
9. Geometry in Weather
   Clouds, rainbow, lightning, moon, sun, tornado, snowflakes, weather maps, weather flags, weather vane, thermometer, barometer, thermograph
10. Geometry in Transportation
    Trains, trucks, automobiles, bicycles, and airplanes
11. Geometry in Insignia
    The use of circles, triangles and parallel lines
12. Geometry in Advertising
    Example of ads using lines, rectangles, circles, semi-circles, parallelograms, curves, and a contrast of several. Also examples of geometric trade marks, copies or originals
13. Geometry in Art
    Non-objective painting, cartoons drawn by the student or clipped from magazines
    Lettering, action figures, Indian art

Pictures showing geometric shapes and principles

14. Symmetry, Congruency, Equality, and Similarity in Everyday Life
    Congruency in manufacturing, e.g. parts of automobiles, ten cent store items, factory made clothes. (Different sizes exemplify similarity, the suit as a whole represents symmetry.)
    Spider web, dandelion, star fish, Easter lily, and snow crystal
    Rug designs, linoleum patterns
15. Geometry in Abstract Design
    Circles, squares, rectangles, triangles, squares, and diamonds present interesting designs if they are mixed together to produce different patterns. In looking at abstract designs you see geometric figures which are usually disguised by additional lines that set the pattern, such as flowers. (Use pictures to show the basic geometric structures of each design.)
16. Geometry in Sailing
    Triangular shaped sails, etc.
17. Geometry in Dressmaking
    Measurements
    How stout people look thin with geometry.
    Effects of pockets pleats, drapes, stripes, double-breasted coats, length of jackets in suits, etc.
18. Geometry in City Planning
    Scale drawing
    Street intersections forming angles
    Plantings in geometric designs
19. Geometry in Church Windows in Our Community
    Colored drawings of various church windows showing use of circles, rectangles, squares and triangles. (One student who was particularly interested and good in art went to a number of churches in town and sketched one window from each church. The sketches were made in pencil and then the various parts colored with crayon. The likeness was so real that it was possible for one acquainted with the church to identify it.)
20. Geometry in Farming
    Machinery
    Contour plowing
    Shapes of fields
    Planting in rows
    Planning or layout of the farm
21. Geometry in Sports
    Hurdles, cross bars, football fields, basketball floors, tennis courts, games played with geometrical objects
    Part played by geometry in training champions in track, pole vaulting or discus throwing
22. Geometry in Nature
    Spider web, snow crystals, honeycombs, cross section of seed pods, leaves, flowers, crystals, butterfly, morning glory, one-celled animals, one-celled plants, natural beauty spots of the world
    Triangular grouping in Easter lily, tulip and dog toothed violet
    Quadrangular arrangement in primrose
    Pentagonal arrangement in hyacinth and tiger lily. (An interesting booklet entitled "You've Been to Geometry-land" shows how Mother Nature used geometry in planning the beauty spots of the world. Full page colored pictures of the following scenic spots were used: a South American landscape taken from a hill, old wheels in a California landscape, a mountain range with a rail fence in the foreground, a scene from the Garden of the Gods, South American power lines, Parliament Building on the Thames, the top of an active volcano, the atomic bomb cloud taken off Bikini in the Pacific, and a famous waterfall. Following each picture there was a diagram emphasizing the geometric arrangements in nature; for example, in the South American scene the trees formed triangles and parallel lines; the mountains in the background, triangles; the peninsula, a circle and a triangle; and the hill in the foreground, a circle.)
23. Geometry in Bridge Construction
    Use of triangle for strength and rigidity
    Circular arc and circular segmental arches
24. Designs in Geometry
    Indian belts
    Sketches of designs based on parallel lines, equilateral triangles, isosceles triangles, squares within squares, stars, crosses, circles, trefoil
25. Geometry in Architecture
    Windows, gables, churches, ancient buildings, modern buildings
26. Geometry in Landscaping
    Parks, roadsides and intersections, homes, public buildings
27. Geometry in Carpentry
    Picture frames, tops for dressing tables, dormer windows, pigpen, roof for barn
    Use of carpenter's square and plane
28. Geometry in Machinery
    Railroads, farm machinery, belts, pulleys, etc.
29. Geometry in Railroads
    Construction of tracks, switches and signals
30. Geometry in Medicine and Surgery
    Pen and ink drawings of instruments and machines
31. Geometry in Churches
    Windows, designs in pulpits, arches, Gothic construction
32. Geometry in Modern Buildings
    Use of parallel lines and rectangles
33. Geometry in Everyday Life
    Curves in eye glasses
    Curved trolley tracks at street intersections
    Examples of planes and other geometric figures
34. Geometry in Millinery
    Drawings or pictures of hats showing effect of lines, symmetry, etc.

35. Geometry in Map Making
    Use of scale drawings
36. Optical Illusions
    Psychology in pictures
37. Making Geometric Figures Using only Straight Lines or Curves in Stitching
    Stitching designs on cards in colors from two basic designs, and the angle and the circle
38. Constructions in Geometry
    Ranging from simple constructions to more difficult ones.
39. Photographs of Uses of Geometry in the Community
    Show use of geometric designs, curves, symmetry and proportion
40. Harmony, Line, Rhythm, and Unity
    Table setting
    Arrangement of furniture
    Selecting drapes
    Designing and choosing clothes to better advantage
41. Geometry in Aviation
42. Geometry in Navigation
43. Geometry in Games
    Chinese checker board, cards, football, basketball, and shuffleboard
44. Geometry in a Camera
45. Geometry in Neckties
46. The Triangle in Everyday Life
    Buildings, bridges
47. The Circle in Everyday Life
    Nature, industry, and home
48. A Day with Geometry
    Follow through a typical day showing contacts with geometry
49. Areas of Plane Geometric Figures
50. Making Models from Erector and Tinker-Toy Sets
    Large models suitable for class demonstration
    Small models suitable for seat work
    Triangles (Isosceles, right, 30–60 right, 45–45)
    Quadrilateral, angle bisector, parallel rulers, protractor with sliding bar, cross staff, center finder, etc.
51. Vocations Requiring a Knowledge of Geometry
52. Reasons for Studying Geometry
53. History of Geometry
54. Great Men in Mathematics
    Pictures of Pythagoras, Archimedes and others with a short biography and their contributions
    History and travels of Pythagoras
55. Application of Triangles to Problems of Rigidity and Circles to Problems of Locomotion
56. Construction of Floor Plans of a House
    Construct plans of cardboard or other material according to scale
57. Compose an Assembly Program to Popularize Geometry
58. Pythagorean Theorem
    Show the mathematics necessary to prove the theorem
    Make an illustrated story including a list of all the theorems, constructions, postulates, axioms and definitions required to prove the theorem
    Construct a model of wood or plastic illustrating proof of theorem
59. Locus in Geometry
    Illustrate the story of an original problem by building a scene of a hidden treasure, etc.
60. Geometry of Flower Arrangement
    Show the use of the triangle, line, symmetry, and balance by diagrams
61. Examples of Ratio and Proportion
    Uses in everyday life
    Newspapers
62. Making a Pantograph
    Explain principle upon which it operates
    Explain its use by series of diagrams
63. Making a Transit or other Surveying Instruments
64. Linkages
    Construction and use
65. Construction of Nine-Point Circle
    Use of paper of at least 12″ by 18″
    Begin on a 60–40 degree triangle
    Colored pencils increase readability of the drawing (The midpoints of the sides, the feet of the altitudes and the midpoints of the segments joining the vertices to the orthocenter of any triangle lie on a circle.)
66. Topology or Rubber Sheet Geometry
    Moebius strip
    Map coloring—only four colors are necessary to distinguish one country from another on a flat map
67. Geometry in the Kitchen
    Food, utensils, measurements, etc.
68. Conflicting Assumptions in Everyday Life
    Analyze reasons and their logical conclusions in common situations of school conduct
    Conflicting assumptions in government, advertising, etc.
69. Types of Reasoning in Geometry Applied to Everyday Life
    Give non-geometric examples of converses, negative analogy, irrelevant facts, special cases, generalizations, jumping to conclusions, fallacious reasoning, and critical thinking
70. Window Transparencies
    Cut original design out of black paper, and use thin paper of various colors to replace cut-out paper. The paper may be oiled or dipped in melted paraffin
71. Paper Folding in Geometry
    Explain a number of geometric principles by folding paper
72. Geometry in the Alphabet
    Show the use of lines and curves in capital letters
    (A very interesting notebook was produced showing the use of the triangle, circle, square, half-triangle, semi-circle, half square and double semi-circle in the

capital letters of the alphabet. First the square was set up as the unit of measure, and its relationship to the other figures mentioned above was shown by means of a diagram; for example, the area of a circle with diameter equal to the side of the square would be .7854 or approximately $\frac{3}{4}$. Then the letter "O" was constructed within a square. Finally the twenty-six capital letters were constructed in simple design with ruler and compass, one on a page with an explanation of each given below; for example; "The letter A is an isosceles triangle with crossbar parallel to the base and dividing the sides proportionally." A similar explanation pointing out the geometric processes used in each was given for each letter.)

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Bradley, Carolyn and Detz, Meredith, *Costume and You.* Scranton, Pa., International Textbook Co., 1949.

(Good for geometry in faces, clothes and fabrics.)

Carson, Byrta, *How to Look and Dress.* New York, McGraw-Hill Book Co., 1949.

(Chapter three shows the use of proportion, harmony, lines and balance in clothes.)

Downer, Marion, *Discovering Design.* New York, Lothrop Lee and Shepard Co., 1947.

(Excellent for showing the use of symmetry, rhythm, line and the abstract in design.)

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(History of geometry)

Estrin, Micheal, *2000 Designs, Forms and Ornaments.* New York, Knickerbocker Publishing Co., 1947.

(An idea and reference book in the development of design)

Gaer, Joseph, *Everybody's Weather.* New York, J. B. Lippincott Co., 1944.

(Excellent illustrations to show the use of geometry in weather)

Hogben, Lancelot, *Mathematics for the Million.* New York, W. W. Norton and Co., Inc., 1937.

(Contains a chapter on what you can do with geometry.)

Huntington, Harriet, *Tune Up.* New York, Doubleday, Doran and Co., Inc., 1942.

(Illustrations of musical instruments)

Kasner, Edward and Newman, James, *Mathematics and the Imagination.* New York, Simon and Schuster, 1940.

Keniston, Rachel P. and Tully, Jean, *Plane Geometry.* Boston, Ginn and Company. 1946.

Morgan, J. McDowell, *Military Medals and Insignia of the United States.* Glendale, Calif., Griffin-Patterson Publishing Co. 1941.

(Excellent illustrations to show the use of geometrical design in metals and insignia.)

Morros, Nelson, "Topology Party," *Science Illustrated,* 1948. March: 20–23.

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(Locus, linkages, how to construct a transit, pantograph, etc.)

National Council of Teachers of Mathematics, Commission on Post War Plans, *Guidance Pamphlet in Mathematics for High School Students.* The Mathematics Teacher, 1947. 40: 315–338.

(Gives vocations requiring a knowledge of all branches of mathematics)

Norling, Ernest R., *Perspective Made Easy.* New York. The Macmillan Company. 1949.

(Emphasizes the use of line, cylinders and

parallels in perspective. Also shows some arrangement of furniture.)

Pingry, Robert E. "A Lesson in Appreciation; The Nine-Point Circle," THE MATHEMATICS TEACHER, 1948. 41: 314–136.

"Psychology in Pictures, "*Science Illustrated*, 1948. March: 57–59.

(Gives optical illusions)

Rockwell, F. F. and Grayson, Esther C. *The Complete Book of Flower Arrangement*. New York, The American Garden Guild, Inc.

(Use of triangles, lines etc. in flower arrangements)

Rosskopf, Myron I., Aten, Harold D., Reeve, William D. *Mathematics, A First Course*. New York, McGraw-Hill Book Co., 1951.

(Application of mathematics to aviation)

Row, Sundara, *Geometric Exercises in Paper Folding*. LaSalle, Illinois, Open Court Publishing Company, 1941.

(87 illustrations of geometric processes by paper folding)

Sanden, Arthur G., "How to Draw the Human Head, Head Proportions and Facial Expressions," *School Arts Magazine*, No. 108, Worcester, Mass.

Seymour, Eugene S., and Smith, James P., *Plane Geometry*. New York, The Macmillan Company, 1949.

(Good for types of reasoning)

Shuster, Carl N. and Bedford, Fred L., *Field Work in Mathematics*. New York, American Book Company, 1935.

(Shows uses of sextant, pantograph and transit)

Simon, Howard, *500 Years of Art in Illustration*. Cleveland, Ohio. The World Publishing Company. 1945.

(Shows the use of geometry in all types of art)

Sykes, Mabel, *Source Book of Problems for Geometry*. Boston, Allyn and Bacon, 1912.

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(Geometry in Nature)

ANNUAL SUMMER CONFERENCE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
San Francisco, California, July 2, 1951

Theme: *Improving the Teaching of Mathematics to Meet the Emergency*
Program: Morning Symposium: *Mathematics and the Present Emergency*
Luncheon: *Get-Acquainted Opportunity*
Afternoon Symposium: *The Training of Mathematics Teachers*
Hotel Reservations:
Hotel reservations will be handled by the NEA Housing Bureau, Room 200, 61 Grove St., San Francisco. Complete information as to available hotels and rates, and an application form appear in the *NEA Journal*. Application can also be made by letter to the NEA Housing Bureau at the above address. In writing please be sure to include:
Preferences of hotels and price range desired
Type of accommodation (single, double bedroom, twin bedroom)
Names and address of all occupants
Dates of arrival and departure
A deposit check, payable to the NEA Housing Bureau, for $10 per room requested
Local Chairman:
Dr. Arthur Hall
San Francisco State College
San Francisco, California

PRESIDENT'S PAGE

YOU WILL be interested in knowing something about the *Cooperative Committee on the Teaching of Science and Mathematics* of the American Association for the Advancement of Science, a committee of some importance and on which the National Council is represented. This committee now numbers fifteen members. Thirteen different national science and mathematics organizations and the United States Office of Education are participating. Among these organizations are: The National Council of Teachers of Mathematics, the Mathematical Association of America, the Central Association of Science and Mathematics Teachers, the National Science Teachers Association, and others from the fields of Astronomy, Physics, Geology, Zoology, Chemistry, and Biology. The National Council of Teachers of Mathematics has been represented on this committee for the past six years. The last meeting of the Cooperative Committee was held in Washington, D. C. at the Federal Security Building on November 17 and 18, 1950, as a part of the Science Education Conference arranged by the Office of Education. This was the second meeting of the Cooperative Committee with the Office of Education and it is planned to hold these meetings annually in the fall. At the first meeting of the Committee with certain members of the staff of the Office of Education in December 1949, the two groups agreed to strengthen the relationships between the Committee and the Office of Education through mutual exchange of information and advice and through joint meetings. The Committee also meets annually in Chicago in the spring.

The AAAS Cooperative Committee concerns itself with the improvement of science and mathematics teaching, as well as with the advancement of the sciences themselves. The Committee has been active for only a few years but their work has been effective. Some of the matters to which they have given attention are: certification of teachers, teacher training programs, the science curricula for college preparation and for general education, laboratory equipment and procedures. The Committee has come out with rather definite recommendations on these and other problems.

Their recent attachment to the United States Office of Education should make more effective the work of the AAAS Cooperative Committee. There will be a material exchange of information and advice on many vital problems such as, mathematics and science in general education at all levels, the early identification of talented youth and the proper educational opportunities for them, the training of mathematics and science teachers, and the problem of adult education in mathematics and science. The fact that the Cooperative Committee plans to hold at least one meeting a year with the Office of Education for the review and evaluation of the activities of both groups, for the discussion of problems of mutual concern, and for cooperative planning is bound to result in making more effective the work of both groups.

For the past two years John R. Mayor has represented the National Council on the AAAS Cooperative Committee. As it now stands, there are three mathematics representatives on the committee: George E. Hawkins, representing the National Council of Teachers of Mathematics; John R. Mayor, representing the Mathematical Association of America; and Donald W. Lentz, Ridge Road School, Parma, Ohio, representing the Central Association of Science and Mathematics Teachers. The former chairman, K. Lark-Horowitz, Purdue University, has found it necessary to resign because of pressure of other duties. The new chairman is Morris Meister, Bronx High School of Science, New York.

H. W. CHARLESWORTH, *President*

ATTENDANCE RECORD OF THE ELEVENTH CHRISTMAS MEETING

University of Florida, Gainesville, Florida
December 27-29, 1950

* Member of the National Council of Teachers of Mathematics.
(E) = Elementary School, (J) = Junior High School, (H) = High School, (JC) = Junior College, (C) = College, (V) = Visitor, (Ex) = Exhibitor, (Su) = Supervisor, (St) = Student.

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*Garrett, Jeannette (H)
Livingston
*Killebrew, H. A. (C)
Tuscaloosa
*Lewis, F. A., Mrs. (C)

ARKANSAS

Arkadelphia
*Young, Ralph W. (C)
Hot Springs
Rapley, John H.

COLORADO

Denver
*Charlesworth, H. W. (H)
Charlesworth, Mrs. (V)

DISTRICT OF COLUMBIA

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*Douglas, Elinor V. (H)
*Lane, Ruth (H)
*Motyka, Agnes (E)
*Schult, Veryl (C)
*Wilson, Dorothy (J)

FLORIDA

Avon Park
*Shearer, Mary C.
Bradentown
*Fitzgerald, Louise (J)
Cherry Lake
Dugan, Russ (Ex)
Clearwater
*Hammock, Ivy (J)
*Johnson, Dorris (J)
Cocoa
*Pomeroy, Jeanette (H)
Pomeroy, W. H. (V)
Daytona Beach
Greer, C. W. (Ex)
DeLand
Foard, Betty (H)
Frostproof
*Webb, Celeste
Gainesville
Awtrey, R. A. (St)
Boles, Ralph C. (C)
*Kidd, Kenneth P. (C)
*Kokomoor, F. W. (C)
*Phillips, Wayland (E)
*Phipps, Dorothy (H)
*Young, Laura (H)
Haines City
*Barrett, Lerlin (H)
Havana
*Evans, Jane (H)
Jacksonville
*Haggard, Margaret
Lakeland
*Brown, Ruth H.
*McIntyre, Maxine
*Mustoe, Anthony Q. (C)
*Reinsch, Bernard P. (C)
Mayo
*Davis, Lera (H)
Miami
*Annin, Elizabeth (H)
*Gibbs, Fletta (H)
*Kimler, Verna (H)
*Mahood, Mildred (J)
*Pierce, Gladys (H)
*Rollins, Annie L. (H)
Miami Beach
*Carlton, Charlotte (H)
*Lyle, Lilla G. (H)
Milton
*Hamilton, Geo. C. (H)
Ocala
*Cromartie, Virginia (J)
*Green, Edna (J)
*Parrish, Josie (H)
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*Echols, Beulah (J)
*McEntire, Evalyn (H)
*Phillips, Theresa (H)
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*Ziegler, Mae (H)
Seabring
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Badger, W. V. (V)
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Johnwick, E. F. (Ex)
*Lindsey, Edna E. (H)
*Patton, Bess (H)
Shipp, John S. (Ex)
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*Wilson, Lottie C. (H)
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Janicki, George (E)
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*Hildebrandt, E. H. C. (C)
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*McCoy, M. Eleanor (H)
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*Sanger, R. G. (C)

KENTUCKY
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*Jones, Edwina (J)

LOUISIANA
Baton Rouge
*Karnes, H. T. (C)
Delhi
*Warden, Meta W. (H)
Lafayette
*Begnaud, Lurnice (H)
*Heard, Ida Mae (C)
*Hoag, Jessie May (C)
*Loflin, Z. L. (C)
McGuire, Norma (C)
Natchitoches
*Maddox, A. C. (C)

MARYLAND
Baltimore
*Herbert, Agnes (J)
Herbert, Mr. (V)

MASSACHUSETTS
Boston
*Betts, Barbara B. (Ex)

MICHIGAN
Ann Arbor
*Jones, Phillip S. (C)
East Lansing
*Blyth, Isobel (C)
Kalamazoo
*Bartoo, G. C. (C)

MINNESOTA
Northfield
*Wegner, J. W. (C)

MISSISSIPPI
Hattiesburg
*Thomas, Ed. J. (C)
Thomas, Mrs. Ed. J. (V)

MISSOURI
Columbia
*Erkiletian, D. (C)

NEW HAMPSHIRE
Exeter
*Adkins, Jackson B. (H)

NEW JERSEY
Montclair
*Mallory, Virgil S. (C)
Mallory, Mrs. Virgil S. (V)
Newark
*Farquhar, Hazel (H)
Trenton
*Shuster, Carl N. (C)
Shuster, Mrs. Carl N. (V)
Union
*Zofay, Agnes (H)
Westfield
*Rogers, Mary C. (J)

NEW YORK
Albany
*Newsom, C. V.
Buffalo
*Gehman, H. M. (C)
*Schneckenburger, Edith (C)
Canastota
*Daniels, Gertrude (H)
Canisteo
*Longley, Russell (E)
Jamestown
*LaVier, Maude A. (J)
New York City
*Alfred, Brother Bernard (C)
*Bakst, Aaron (C)
Bakst, Mrs. Aaron (V)
*Fehr, Howard F. (C)
Fehr, Mrs. Howard F. (V)
*Higgins, William B. (C)
*Jablonower, Joseph
Niagara Falls
*Horn, Ruth
Syracuse
*Farndacher, Karl (St)

NORTH CAROLINA
Durham
*Rankin, W. W. (C)
Rankin, Mrs. W. W. (V)
Greenville
*Phillips, Orval L. (C)
Marion
*Alley, Catherine (H)
Wilmington
Morris, Texys (H)

OHIO
Columbus
*Lazar, Nathan (C)
*Schacht, John F. (H)
Granville
*Rupp, E. C. (C)
Kent
*Brumfield, Ema Lou (C)
*Stapleford, Edward (C)
Medina
*Insprucker, J. H. (H)
Oxford
*Christofferson, H. C. (C)
Christofferson, Mrs. (V)

OKLAHOMA
Stillwater
*Zant, James H. (C)
Zant, Mrs. James H. (V)

OREGON
Portland
*Hoel, Lesta

PENNSYLVANIA
Bala-Cynwyd
*Lamb, Beatrice (J)
Elizabethtown
*Heilman, C. E. (C)
Heilman, Mrs. C. E. (V)
Kingston
*Jones, Freda
Philadelphia
*Wyatt, Ruth
Uniontown
*DiPrampero, Louise (H)

RHODE ISLAND
Providence
*Bennett, Albert (C)

SOUTH CAROLINA
Columbia
*Dicks, A. Lois (C)
Rast, Dorothy (St)
Greenville
*Werts, Lily Mae (H)
Rockhill
*Rogers, Hortense (C)
Silverstreet
*Werts, Eoline (E)

TENNESSEE
Knoxville
*Brown, Kenneth E. (C)
Brown, Mrs. K. E. (V)
Nashville
*Banks, J. W. (C)
*Miser, W. L. (C)
Union City
*Ray, Vera (H)

TEXAS
Dallas
*Fulton, Will P. (H)
Huntsville
*Lane, Ruth O. (C)
Smith, Lolie (C)
Nacogdoches
*Layton, W. I. (C)
San Antonio
*Rees, Dorothy (H)
San Marcos
*Cude, Don (C)

VIRGINIA
Arlington
*Burington, Artha (H)
Charlottesville
*Lankford, F. G., Jr., (C)
Lankford, Mrs. (V)
Lexington
*Knox, R. H. (H)
*Purdie, K. S. (H)

WASHINGTON
Seattle
*Haynes, Helen (J)

WEST VIRGINIA
Montgomery
*Nolan, Irene (C)
Morgantown
*Dorsey, Catherine (H)
St. Albans
Holmes, Margaret F. (H)
*Lynch, Kathryn (H)

WISCONSIN
Madison
*Mayor, John R. (C)
Racine
*Potter, Mary (Su)

CANADA
Toronto
*Petrie, P. A. (C)
Petrie, Mrs. P. A. (V)

CUBA
Havana
*Rivas, Silvia (H)
*Rodriques, Maragrita(H)

(*Continued on page 244*)

SIGNS OF PROGRESS OF THE AFFILIATED GROUPS PROGRAM

By JOHN R. MAYOR
Chairman Committee on Affiliated Groups

BY THE time these paragraphs are in print the Second Delegate Assembly will be past history. Early reports indicate that the Second Delegate Assembly, like the First in Chicago, should be an important event in the history of the National Council. In this Second Assembly nearly fifty delegates, representing members of Affiliated Groups, including some 5,000 members of the National Council and more than 10,000 teachers of mathematics, will have debated policies of the National Council and made decisions important for the future of the Council.

Members of the Affiliated Groups should start immediately to plan for the Third Delegate Assembly. These meetings can be most valubale if the local groups provide adequate opportunities for their members to express opinions on all issues debated. Your Group should give consideration throughout the year to ways and means of making the National Council, with the assistance and cooperation of its Affiliated Groups, a more effective agency for the improvement of mathematics instruction at all levels and in all parts of the country. There has never been a time in the history of American education when teachers of mathematics faced a greater challenge and a greater opportunity.

Following conferences at the Florida meeting of the National Council in December plans are being developed for a number of new activities to be sponsored through the Affiliated Groups. These plans include the organization of a Speakers Bureau on which a preliminary announcement was made in the December issue of THE MATHEMATICS TEACHER; a membership campaign, which should benefit both the National Council and the Affiliated Groups, and which is timed so that new members may take advantage of the special reduced price offer on the 19th and 20th Yearbooks; a cooperative investigation of some critical problems in mathematics education; and the organization of a traveling exhibit which may be shared by various Affiliated Groups.

Participation of the Affiliated Groups will, of course, be on a completely voluntary basis. Some of the larger Groups and those which have been longer established may wish to participate in all of these activities. Other groups may choose to take part in only one of these activities, or perhaps none for the first year. Naturally the Groups will be free to determine their own organizational set-up for carrying on this work. In many cases it may be desirable for the Affiliated Groups to organize coordinating committees to work in conjunction with the Committee on Affiliated Groups as proposed by the First Delegate Assembly. It is in the best interests of all concerned that as many different teachers as possible be brought into this work and that in no case a "chosen few" have all the privileges or all the burdens involved in active participation in Group work.

As plans materialize they will be reported in this section of THE MATHEMATICS TEACHER. In the development of plans the Newsletter of Affiliated Groups will be used as a means of communication. One of the purposes of early announcement of plans in this section is to give all readers of THE MATHEMATICS TEACHER an opportunity to make suggestions during the developmental stages and to offer assistance and cooperation to their own Affiliated Groups or to the national committee.

In the May number of THE MATHEMATICS TEACHER, in this section, an analysis will be made of the organization of the various Affiliated Groups with par-

ticular reference to the cooperation of city and county groups, within a state, with a state organization. This report is to be made upon special request.

New Groups in Philadelphia and in Pinellas County

The Association of Teachers of Mathematics of Philadelphia and vicinity is the first Group to apply for affiliation with the National Council in 1951. In the letter from M. Albert Linton, Jr., of the William Penn Charter School and president of the Association, which brought the good news that the Philadelphia mathematics teachers wished to become affiliated with the National Council was this sentence:

"It is my sincere hope that affiliation with the National Council will serve as an incentive to us to try to improve our services to the mathematics teachers of Philadelphia and suburbs." Since this so well expresses also the sincere hope of the Committee on Affiliated Groups for each and every Affiliated Group it is repeated with great appreciation to Mr. Linton.

At the Florida meeting, the attention of the Chairman of the Committee on Affiliated Groups was called to the omission of the Pinellas County (Florida) Council of Teachers of Mathematics from the list of Affiliated Groups in The Mathematics Teacher for December. The Chairman apologizes for this error and is happy to welcome Pinellas County to the family of Affiliated Groups especially after the fine contributions of teachers from Pinellas County to the Christmas meeting in Gainesville. All who attended the Florida Meeting are grateful for the hospitality shown by all of the Affiliated Groups in Florida which cooperated in making that meeting a success.

A Ten Year Record of 100 Per Cent Membership

The A.T.M.N.E. Newsletter for November, 1950 includes the following announcement:

"The Roxbury Latin School has had a 100 per cent membership in the A.T.M.N.E. for the past ten years. Can any other school equal or exceed this claim?"

This question, addressed to members of the Association of Teachers of Mathematics of New England, is repeated for all other Affiliated Groups. Can other Affiliated Groups point to examples of such long and loyal support?

Attendance Record, 11th Christmas Meeting

(*Continued from page 242*)

ATTENDANCE RECORD

State	Members	Guests	Total
Alabama	3	0	3
Arkansas	1	1	2
Colorado	1	1	2
District of Columbia	6	0	6
Florida	60	10	70
Georgia	11	6	17
Illinois	11	3	14
Indiana	5	1	6
Iowa	2	0	2
Kansas	2	0	2
Kentucky	1	0	1
Louisiana	7	1	8
Maryland	1	1	2
Massachusetts	1	0	1
Michigan	3	0	3
Minnesota	1	0	1
Mississippi	1	1	2
Missouri	1	0	1
New Hampshire	1	0	1
New Jersey	5	2	7
New York	13	2	15
North Carolina	3	2	5
Ohio	7	1	8
Oklahoma	1	1	2
Oregon	1	0	1
Pennsylvania	5	1	6
Rhode Island	1	0	1
South Carolina	4	1	5
Tennessee	4	1	5
Texas	5	1	6
Virginia	4	1	5
Washington	1	0	1
West Virginia	3	1	4
Wisconsin	2	0	2
Canada	1	1	2
Cuba	2	2	4
Totals	181	42	223

KNOW YOUR NATIONAL COUNCIL REPRESENTATIVE

By KENNETH E. BROWN
University of Tennessee, Knoxville, Tennessee

MISSOURI REPRESENTATIVE

MISS ELLA MARTH is a native of Missouri where she was granted three degrees, including the Ph. D. from St. Louis University. The special work in graduate mathematics and the 13 years as a classroom teacher in the elementary schools of St. Louis have provided her with a rich background of experience that makes her particularly fitted for her present position of the training of future mathematics teachers. In addition to her duties as a faculty member of Harris Teachers College and consultant in arithmetic for St. Louis Public Schools, she spends many hours in giving publicity to the work of the National Council. Her efforts have been rewarded by a substantial gain in membership in Missouri since she became State Representative in 1947.

MISS ELLA MARTH

MISS EUNICE LEWIS

OKLAHOMA REPRESENTATIVE

MISS EUNICE LEWIS was born in Tennessee, however her high school and college education was received in Oklahoma. Her experience in teaching in the high schools of Covington, Sapulpa, and Tulsa has given her a knowledge of high school problems that is invaluable in her present position of training prospective mathematics teachers at the University of Oklahoma. As Supervisor of Mathematics at the University High School she keeps in close touch with high school students and their problems. The mathematics teachers indicated their high regard for the professional zeal of Miss Lewis by electing her President of the Oklahoma Council of Teachers of Mathematics. The membership of the National Council in Oklahoma has increased more than fifty per cent during the last four years under her guidance.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

MODELS MADE FROM MAKIT TOY

THE following devices are easily constructed of MAKIT TOY, a colorful, sturdy, building set manufactured by the W. R. Benjamin Company of Granite City, Illinois. The set retails in three sizes, PRIMARY, SUPER, and JUMBO MAKIT. The JUMBO size costs about $2.50, contains 275 pieces, and is sufficiently complete to permit the simultaneous construction of two or three working models. Directions for construction in this article make direct reference to the names of the parts included in a set.

Angle Demonstration Device

Two colored wheels, a metal bushing, a collar, a short red rod, and three plain rods are needed to construct this device. (See Fig. 1.) Red rods fit loosely in wheel holes; plain rods fit tightly. Red rods capped with metal bushings also fit tightly.

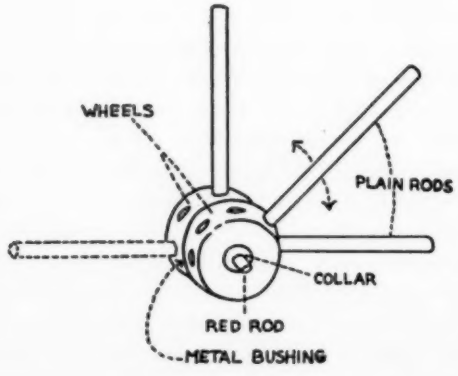


FIG. 1

Insert the red rod capped with the metal bushing into the center of one of the wheels. Slide the red rod through the center of the other wheel, and push the collar down over the exposed end of the red rod. The result is a potential vertex of the angles to be constructed. It may be designated as the "red vertex," the "yellow vertex," or the "red-blue vertex" depending on the choice of colored wheels.

Insert the two plain rods in the holes of one wheel at right angles to each other. Fix the third plain rod in the other wheel. The finished product is a right angle with a movable third line which may be used to show how angles are generated, to illustrate the concept that a degree is a unit of revolution independent of the length of the sides of an angle, to demonstrate the inverse variation of complementary angles, and to illustrate the definition of complementary angles.

The device can be adapted to supplementary angles by fixing the plain rods in the holes so as to form a straight angle. (Note the position of the rod drawn with dotted lines.)

Triangle Demonstration Device

To build this device, construct three vertices, each of a different color, by the method described above. Choose three plain rods and insert them in the wheel holes, making sure that no two rods are inserted in the same wheel. (See Fig. 2.) Keep one end of the third rod free for the purpose of demonstrating to the class that each angle varies freely. By fixing the third rod in the remaining wheel, the rigidity property of a triangle may be demonstrated, and also the corresponding

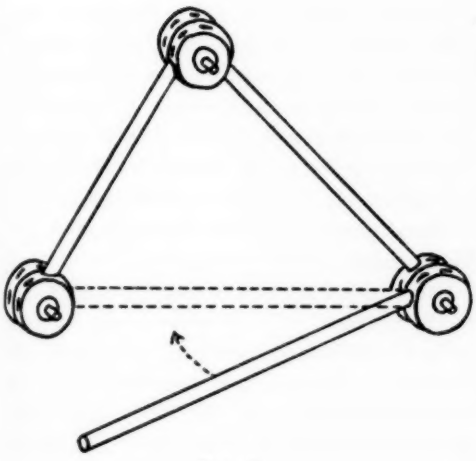


FIG. 2

fact that triangles are determined by the lengths of their sides.

The rods in MAKIT TOY are so proportioned that two of one length and one of the next longer will form an isosceles right triangle. Also, combinations of short rods with joining wheels are equal to longer rods. Thus, models illustrating similar triangles may be built quickly during class time by students themselves.

General Quadrilateral Device

One more "wheel vertex," two equal long rods, two equal shorter rods, and two other rods of sizes different from the two pairs mentioned are required to transform the triangle demonstration device into a general quadrilateral device. (See Fig. 3.)

Beginning with four plain rods of different lengths, it is possible to illustrate the evolution of the various types of quadrilaterals by substituting rods in accordance with the special conditions of the definition of each figure. The parallelogram can be continuously deformed to show that no matter how the angles are permitted to vary, the sides remain parallel and equal.

By inserting the rods at one vertex into a single wheel at right angles to one another, the principle that a rectangle is a parallelogram with one right angle may be demonstrated.

The possibilities for constructing working models of MAKIT TOY are limited only by time and imagination, and the models are adaptable to classes in both the junior and senior high school.

Nona Mary Allard
Cromwell Public Schools
Cromwell, Minnesota

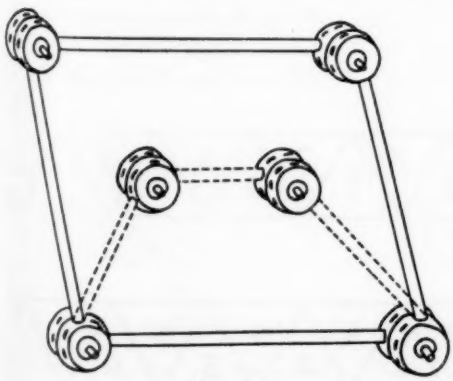


FIG. 3

HEXAHEXAFLEXAGRAMS

Editor's Note. This article is being published in response to several special requests. After reading it you'll be as excited about it as the people who asked to have it printed. Upon receiving the manuscript the editor of this department couldn't wait to get his hands on a suitable strip of paper and produce one of "those things." The directions are remarkably clear and easy to follow—a tribute to the ingenuity of Miss Margaret Joseph who submitted the article. High school students will enjoy making the hexahexaflexagram and "flexing" it.

To make the hexahexaflexagram, obtain a piece of adding machine paper about 28 inches long. The exact length needed will depend on the width of the strip used.

Fold one end forming a crease about $1\frac{1}{2}''$ long. See Figure 4. Then fold the paper so that point B falls along crease CD

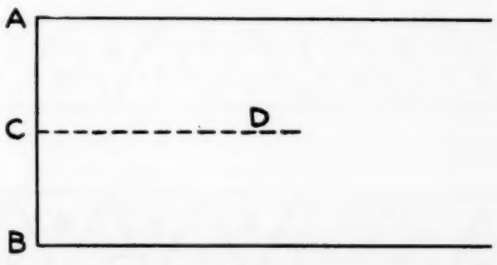


FIG. 4

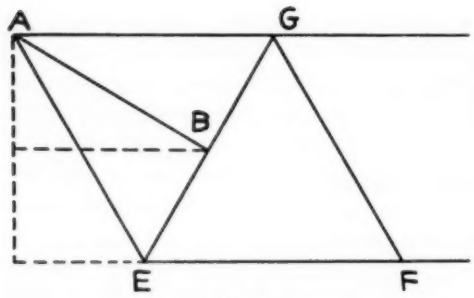


FIG. 5

forming triangle *ABE* as shown in Figure 5. This triangle will be a 30–60 degree right triangle with the 60-degree angle at *E*. Now fold *AE* along lower edge *EF* to form an equilateral triangle *EFG*. This and all subsequent foldings must be done carefully so that good triangles are formed. If carelessly made, there will be a flat space instead of a vertex for the top of the triangle.

Since the first triangle was formed by folding the paper forward, the second should be made by folding the paper under or away from you so that the triangles are made by repeatedly folding back and forth until 19 equilateral triangles have been made. Cut off the excess that remains as well as the first little right triangle that was made. Then lay the strip flat on a desk and number as shown in Figure 6. The last triangle will be a blank. Turn the strip over so that the end marked *A* will remain on your left. Number this side as shown in Figure 7. If a different colored pencil is used for each number the entire figure will be more attractive.

Now the strip must be folded. There is a rhythm to the folding which becomes apparent after you have made a few folds. Begin by holding the strip in the position *CD* shown in Figure 7. Fold the second triangle numbered 4 over the first one numbered 4, then fold triangle 5 on triangle 5 in the same manner, and triangle 6 on triangle 6. (You may have to look under the strip at times to see the numbers desired.)

Stop now and check your progress. In reading from left to right you should have the following arrangement: a blank triangle, triangle 3, triangle 1, and a second triangle 1 on top. If you do not have this arrangement recheck the directions already given.

Next fold triangle 1 on triangle 1, and another triangle 3 will appear on top. The series of folds made to this point are repeated twice more and then the folding is complete. However, when you come to the last arrangement you will have a blank triangle, and a triangle 1 remaining at the right end of the strip or hexahexaflexagram. Put the blank triangle from the extreme left over the triangle numbered 1. (You will note that there is a 1 on the underside of the blank triangle.) Finally paste the two blank triangles together. When the paste is dry the hexahexaflexagram will easily open or flex so that the same number will always appear on all triangles. Hold the figure towards the bottom and do not force it. If it will not open where you are holding it, change the position of your fingers and try again. All six numbers can be made to appear.

MISS MARGARET JOSEPH
South High School
Milwaukee, Wisconsin

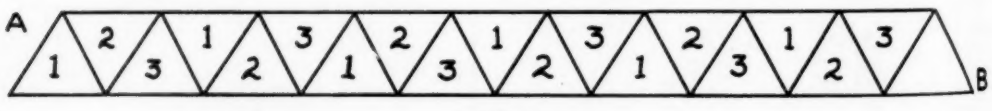


FIG. 6

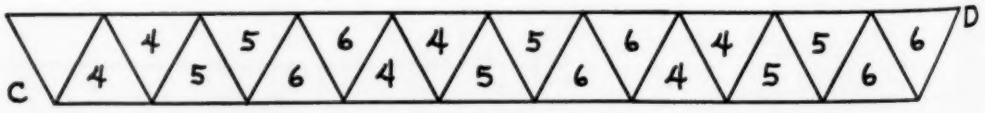


FIG. 7

A Solid Geometry Demonstration*

A physical principle familiar to nearly every high school student is that the intensity of illumination varies inversely as the square of the distance from the source of light. The existence of this variation can readily be verified by using appropriate science laboratory apparatus. However, it is much more interesting and satisfying to use essentially the same demonstration materials and examine the problem from the standpoint of the mathematical principles involved.

The apparatus to be used consists of a candle, a yardstick, two wooden triangular shaped supports, one opaque cardboard screen 1 ft. square, and four smaller screens. The central parts of the smaller screens should be cut out. Figures to be cut out may include a circle, triangle, square, and pentagon. Either the cut-out screens or the figures cut out may be used in the demonstration. In the first case the base of the figure (to be described below) will be an illuminated area; in the second case it will be a shadow.

The apparatus is set up as illustrated in Fig. 8. The large screen (Y) and one of the small screens (X) may be fastened to the triangular wooden blocks with thumb tacks. The screens and the candle are then placed on top of the yardstick as shown in the diagram. The planes of the screens must be parallel to each other and perpendicular to all long edges of the yardstick. When the candle is lighted an area outlined as triangle A′B′C′ will be illuminated on screen Y. For best results the demonstration should be conducted in a darkened room. Now let P represent the vertex of a pyramid with triangle $A'B'C'$ as its base. LO' will be equal to the altitude of this pyramid. Triangle ABC (the area cut out of screen X) will be a section of this pyramid, and LO will be equal to the length of the perpendicular from P to the plane of triangle ABC.

By direct application of a solid geometry theorem and its corollary we obtain the following relation:

$$\frac{\text{Area of Triangle } ABC}{\text{Area of Triangle } A'B'C'} = \frac{(LO)^2}{(LO')^2}.$$

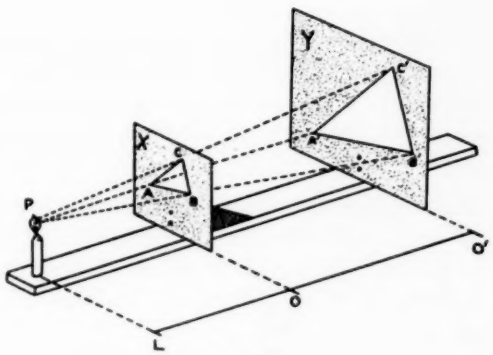


Fig. 8

Theorem. If a pyramid is cut by a plane parallel to its base: (a) The lateral edges and the altitude are divided proportionally. (b) The section formed is a polygon similar to the base.[1]

Corollary. A section of a pyramid parallel to the base is to the base as the square of its distance from the vertex is to the square of the altitude of the pyramid.

Substitution of the small screens with the square, circular, and pentagonal cut-outs will help add a note of generality to the result demonstrated with the triangular pyramid. In the case of the circular cut-out it should be pointed out that a theorem and corollary similar to those stated above for the pyramid can be stated for a cone.

The connection between the mathematical principles just described and the principle concerning illumination stated earlier needs only a little clarification. From the theorem and its corollary it is apparent that the area illuminated increases as the square of the distance from the source of light. By the law of conservation of energy we know that the amount of light passing through the cut-out screen X is the same as that illuminating the large screen Y. Hence the intensity of illumination varies inversely as the square of the distance from the source of light.

* The demonstration described in this article was developed in The Mathematics Laboratory, Monroe High School, St. Paul, Minnesota.

[1] This statement of the theorem and the corollary which follows were taken from the textbook, *Modern-School Solid Geometry*, by Smith, Rolland R., and Clark, John R., World Book Company, Yonkers-on-Hudson, 1939.

RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
School of Education, New York University, New York 3, New York

The Questions: How do students differ in what they do in attempting to solve problems? How can a teacher use his knowledge of these differences to improve general problem-solving ability?

The Study: Bloom, B. S. & Broder, L. J. *Problem-solving Processes of College Students.* University of Chicago Press. Chicago 37, Illinois. 1950.

The teacher of mathematics who sees evidence that his efforts are accompanied by an improvement in the general problem-solving ability of his students probably feels that he is making a major contribution to both general and special education. This exploratory study is a rich source of suggestions of ways of bringing about this much-desired result.

It is tempting to assume that the problem of problem-solving is the same for all students and, therefore, to infer that it is sufficient to standardize the teaching of problem-solving by patterning it after some such analysis as Dewey's famous five steps described in his book. "How We Think." To most of us it is not news that the fact of individual differences spoils this happy illusion. This study makes that reality even clear.

It is also tempting to believe that a study of the written work of students in doing problems gives all the clues needed for wise and adequate guidance in improving problem-solving ability. This isn't so, either. Hence, it is not surprising to learn that in this study there is used the *method of making a record of what the student says as he thinks aloud in attacking his problems.*

The purpose of the first part of the study was to discover the kinds of ways in which students differ in solving problems. One group of students consisted of six individuals with high scholastic aptitude test scores and with records of A's and B's on achievement examinations given at the College of the University of Chicago; the second group of six students had low aptitude test scores and achievement test grades of *D* and F. These two groups were designated the "successful" and "unsuccessful" problem-solvers respectively. Each of the twelve students worked with twenty problems selected from a variety of fields, such as science, social studies, English and mathematics. These problems were such that memory and puzzle aspects were reduced to a minimum; they were characterized by clear-cut goals and open to definite methods of attack. Each individual student attacked these problems in several sittings in a private conference with one interviewer. As he did so the *interviewer recorded what the student said as he thought aloud about the problems.*

The major differences between the two groups were found to fall into five general categories: understanding the nature of the problem, understanding the ideas in the problem, the general approach to the solution and the attitude toward solutions. The first of these involved difficulties in starting the attack upon the problem and in solving the problem as presented. The second revealed differences in the ability to bring relevant knowledge to bear upon the problem and in the capacity to realize the implications of ideas in the problem. The third showed differences in aggressiveness of attack, in the care and system in thinking about the problem and in the ability to follow through on a process of reasoning. The last uncovered differences in the faith placed in reasoning versus guessing, in the amount of confidence in attacking problems and in the tendency

to introduce personal reactions not implied by the problem.

These characteristics, which varied in these two extreme groups, were checked by employing a group of twenty-seven typical students. The same traits appeared. The major difficulties of this group were in trying to decide what to do, in maintaining an objective attitude about the problems, in reasoning logically and systematically, in completing a chain of reasoning and in bringing to bear relevant subject matter which apparently was known. Six to eight weeks later these students exhibited more carefulness and system, increased confidence, more thinking and, as might be expected, more knowledge of the subject matter involved. It seems apparent that these traits were related. It was noted that there was no change in objectivity toward the problems. Dislike, fear and personal and social values associated with the problems previously were still strong.

Having isolated the differences in the characteristics of the students in solving problems the experimenters next studied the kinds of differences in the problems themselves. Some of these of a trivial nature involved difficulties with the format of the problem, the method of indicating answers, the presence of multiple choice items having negative phraseology and the appearance of relative terms like "frequently" and "usually." The difficulty of the problems also varied with the ease of translating the ideas and terms of the problem into more familiar or more specific ones and with the possibility of reducing a problem to its essentials, bereft of excess verbiage.

Finally, two remedial programs were carried out. The first was an individualized one; the second involved procedures with groups of five to twenty students. The basic assumption was that each individual has his own peculiar cluster of problem-solving characteristics which had grown up with him over a considerable period of time. The criteria employed in selecting the problems for the programs were (1) that a small amount of specific information was needed, (2) that they were the kind usually met in the college examinations and (3) that they should differentiate between good and poor problem achievers. Next, the students were oriented and motivated for the remedial program. They were told, among other things, the principal aim was to produce an improvement in *general* problem-solving ability, that success would depend upon the effort exerted and the development of confidence, and that practice sessions would be essential. The remedial procedures consisted of a determination of the individual problem-solving characteristics by recording the "thinking aloud" comments of the students in two sessions of one and one-half hours. In the next session students were encouraged to analyze their methods and compare them with those of very successful problem-solvers. In the remaining sessions further analysis was made and practice in applying good problem-solving methods to new problems was provided. To facilitate these procedures a pamphlet of sample problems and a printed check list of problem-solving characteristics were made available for student use.

In the experiment with the groups of five to twenty the early sessions were the same as in the individualized approach. After the third session a sample problem would be flashed on a screen. A student volunteer would then give his solution for analysis and criticism by his fellow students. Eventually, the solution would be compared with a model solution for the general methods of problem-solving involved. During the practice sessions which occurred twice a week it was necessary and desirable to divide the large group into smaller groups of two or more students. In each of these the students would take turns presenting their solutions to the members of their small group for analysis and comparison with a model solution. At times the problems

attacked were centered about one kind of subject matter; at others the emphasis was on general methods of problem-solving.

These remedial programs were evaluated by the students themselves, by certain faculty members, by an analysis of the results of a problem-solving test and by a study of the improvement in examination results over the experimental period. The first three methods revealed improvement on the part of a varying number of students in a variety of aspects of problem-solving. One of the most frequent results was an increased confidence in problem-solving. The examination results showed that the gains made by the experimental groups of students were statistically significant to a high degree while the gains of other control groups of comparable ability and scholastic success were inconsequential.

CORRECTION: We regret that on page 153 of the February issue the heading "Annotated References to Research Studies" was inadvertently omitted above the references to Dr. Kramer's study. Readers may have inferred that the study was considered trivial because of the brief comment but we did not intend that such an implication be drawn.—J.J.K.

APPLICATIONS

Edited by SHELDON S. MYERS
University School, Ohio State University, Columbus, Ohio

CONCERNING Ar. 5 Gr. 7–9 "Famous Explosions on the Earth," Dr. John Kinsella writes as follows: "The January MATHEMATICS TEACHER arrived today. Your department pages came into view. There was something on 'famous explosions.' That reference brought back memories of University School where I tried hard to make 7th and 8th grade mathematics function in science materials. I'm not sure the credit for the six problems belongs to me; I do know that the content was provided by Dr. Charles C. Weidemann who was teaching there when I came in September 1942. I may or may not have provided the questions on the material." In giving credit your editor used the evidence of the initials "JJK" in the corner of the guidesheet. Therefore, it is likely that Dr. Kinsella made up the questions based on materials gathered by Dr. Weidemann.

Al. 6 Gr. 9–10 TRANSLATING STATEMENTS INTO ALGEBRAIC FORMULAS

Paul J. Eby, mathematics instructor in General College, Boston University, sent us the following problems about which he wrote: "I use them to give practice in symbolizing verbal relationships and solving simple equations. The following problems[1] may provide interesting sidelights for a class in elementary algebra. An excellent way to use them is to present them as they are and ask the pupils to write a formula to represent the statement. When this is done, they may collect their own data and solve the problem for the desired unknown."

1. How much wall paper do you need to redecorate a room?

This is one of the shortcuts of the trade: measure the square footage of the walls you want to paper. Divide by 36. The result will be the number of single rolls you'll need.

Most paper is sold in double rolls, so divide your result by 2 for the number of double rolls needed. (Deduct 1 single roll for every 2 ordinary size windows or doors and each 50 square foot opening.)

2. How much paint will you need?

For a room: Multiply the width of each wall by its height in feet. Add these and divide by 600. The answer is the number of gallons you'll need for one coat.

For the outside of your house: Figure as above but divide by 400. The answer is the

[1] Compiled by Churchill, Judith Chase, "A Woman's Tricks," *The Woman's Home Companion*, March 1950, pp. 32–33. Copyright 1950, Crowell-Collier Co. Reprinted by permission from the publishers.

number of gallons you'll need for two coats of outside paint.

For porches and floors: Multiply the width by the length and divide the result by 250 for two coats of paint.

3. How much wax will you need for your floor?

Measure the length and width of your floor in feet, and multiply the two numbers.

For liquid wax: Divide the answer by 300 for the number of quarts you'll need.

For paste wax: Divide by 400 and the answer will be in pounds of wax needed.

4. How much cloth does a woman need to make a dress?

The following directions apply for an average dress being cut from silk, rayon, or cotton material which averages 36 to 39 inches in width.

Measure in feet your height from the neck down. To this, add the length of your arm from shoulder to finger tips. Multiply by 2 and divide by 3. This will be the amount of material you'll need in yards.

In the light of the above directions, is the pattern a factor in the amount of cloth needed?

5. What is your proper weight?

First of all, run a tape measure around your chest at the level of the end of the breast bone. Now exhale. Now measure your chest circumference, pulling the tape measure quite tight.

Next, divide this measurement by 4. Take this answer and multiply it by itself. Take the result and multiply by your height in inches (without shoes). Divide by 27.69. The answer is your normal weight. (From a University of Illinois formula based on body build.)

6. How old are your trees?

Measure the tree trunk's diameter in inches at chest height. Multiply by the number below which corresponds to the tree. The answer is its age.

White elm, tulip, chestnut	2½
Black Walnut	3
Black Oak	3½
Birch, sweet gum, sycamore, chestnut, oak, red oak, scarlet oak	4
Ash, white ash	5
Beech, sour gum, sugar maple	6
Shagbark, hickory	8

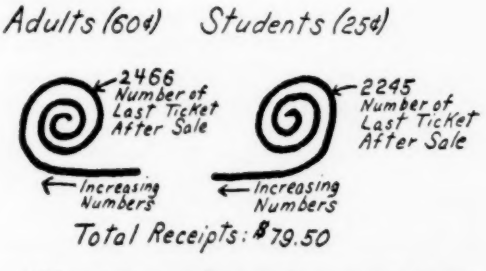
Al. 7 Gr. 9–11 ALGEBRA RESCUES A STUDENT TICKET SELLER IN DISTRESS

We are indebted to Eugene P. Smith for the following real-life anecdote. Mr. Smith teaches mathematics at the University School, Ohio State University.

One morning before school a most distraught, young high school girl rushed into Mr. Smith's office and woefully exclaimed: "Oh, Mr. Smith, what will I do! I forgot to take down the number where I broke a roll of tickets into two rolls—one for adults and one for students, and now I don't know how many of each kind I sold. I simply must find out, so that I can fill in the tax form!"

"Well, Mary," said Mr. Smith calmly, show me all the data you have on your ticket sale last night. Perhaps we can use algebra to find out."

With this encouragement, Mary spread the following sketch of the data before Mr. Smith.



"Now let's see, Mary. Can you find out how many tickets you sold altogether?" asked Mr. Smith.

"Why sure. I subtract 2245 from 2465 and get 220 tickets sold."

"Now, according to your figures, you know the total number of tickets sold, the price of each kind, and the total amount of receipts. Does that remind you of anything?"

With a squeal of delight, Mary exclaimed, "Why that's just like some of those verbal problems we've been having lately in algebra! Hmm. If I let "s" be the number of student tickets and "a" be the number of adult tickets, I could write the equation: $s+a=220$. But I can't find s and a from that equation. We always had two equations when we had two letters. I wonder if the prices and receipts would help any? I've got it! The money we took in on student tickets could be represented by $.25s$ and the money from adult tickets by $.60a$. The sum of these would be my total receipts. I can now write the other equation: $.25s+.60a=79.50$."

Mary then wrote down the two equations and proceeded to solve them as follows:

$$.25s+.60a=79.50$$

$$s+a=220 \qquad s=220-a$$

$$.25(220-a)+.60a=79.50$$

$$55-.25a+.60a=79.50$$

$$.35a=24.50$$

$$a=\ 70 \text{ adult tickets}$$

$$s=220-a=150 \text{ student tickets}$$

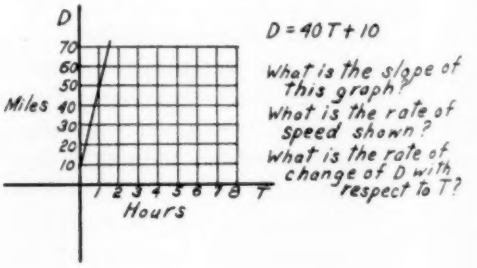
After checking these figures with prices to see if they gave the correct receipts, Mary dashed happily off to the business office to fill out the tax form.

Al. 8 Gr. 9–12 Some Well-Known Rates Illustrated in Algebra

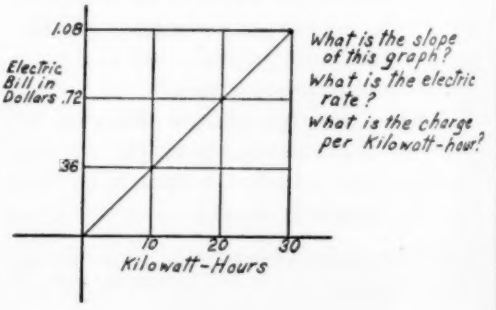
The word "rate" must be understood physically and algebraically for an adequate grasp of the function concept in algebra. The slope-intercept concept of the linear function can be successfully taught in beginning algebra and can provide a powerful motivation for students studying the subject. The word "rate" can early be associated in student thinking with the slope "m" in the general form: $Y=mX+b$. "m" is variously defined for the student as $\Delta Y/\Delta X$ and as the change in the vertical variable divided by the corresponding change in the horizontal variable. If Y increases as X increases, the slope is positive; if Y decreases as X increases, the slope is negative. Some of the sophisticated students might define the slope "m" as the tangent of the angle that the linear graph makes with the horizontal axis. The word "rate" might also be defined verbally as follows: "The rate of change of variable A with respect to variable B is the amount A changes for a unit change in B." The values of physical rates, therefore, depend on the dimensions in which the variables are measured.

It is our opinion that the foregoing ideas could not be taught adequately unless they were developed in the context of many illustrations within the experience of high school boys and girls. Following are some illustrations of rates used by the writer for a number of years in beginning algebra.

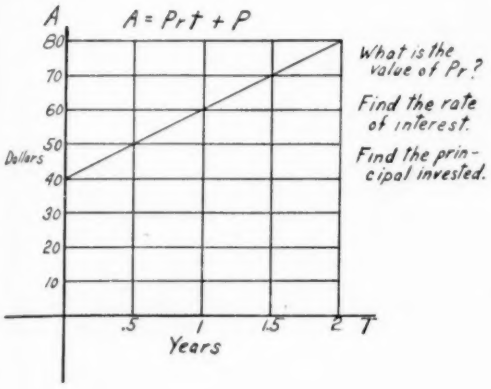
8.1 Car Moving at Constant Speed



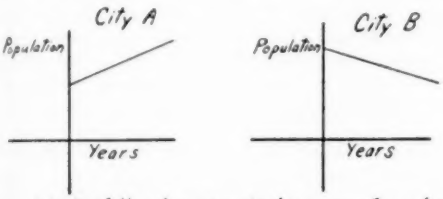
8.2 Your Electric Bill



8.3 Growth in Principal Invested at Simple Interest

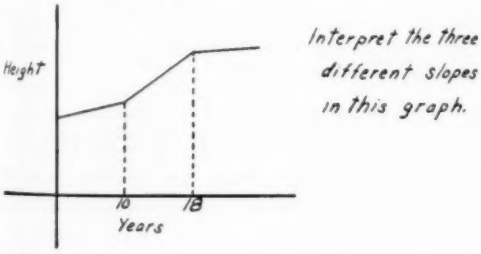


8.4 Population Change in Two Cities

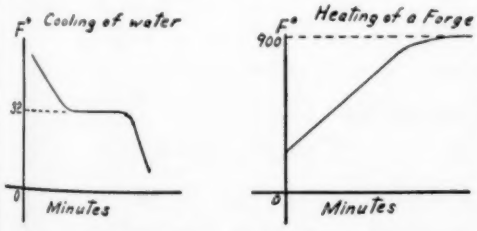


Which of the above graphs has a positive slope and which a negative slope? What is the interpretation of positive and negative slopes in the graphs above?

8.5 Growth of Jim

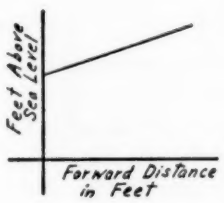


8.6 Cooling of Water **8.7 Heating of a Forge**

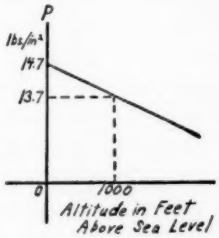


What is the meaning and cause of the zero slope in the left graph? Why does the slope on the right graph reduce to zero at 900° F?

8.8 Climbing Airplane

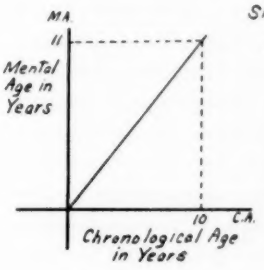


8.9 Change in Air Pressure With Rise in Altitude



Interpret the slopes of the two graphs above.

8.10 The IQ, or Intelligence Quotient



$\text{Slope} = \frac{\Delta M.A.}{\Delta C.A.} = \frac{11}{10} = 1.1$

$1.1 \times 100 = 110\%$ or IQ of 110

The IQ, or Intelligence Quotient, has been defined as the per cent the M.A. is of the C.A. Here it can also be interpreted as a rate of mental growth.

While the above approach to the topic of rates can make a significant contribution to the general education of students, it also lays a sound foundation for the understanding of interpolation with logarithms and trigonometric functions and for the understanding of the delta process in calculus.

Sister Mary Agnes of Central Catholic High School in Great Falls, Montana, requested the solution to "P. 1 A Problem For Thinkers" in the November issue, which we have neglected to give.

Referring in parentheses to the numbered statements: the father (5) is not the instructor because the instructor has no living children (6) and is not the president because of (3). Therefore the father in (5) is the professor but cannot be named Mr. Haines or Mr. Jones because he would not confuse his own son (5). The professor is not Mr. Ross because of (1). Therefore the professor is Mr. James. If the president is not Mr. Jones (2, 3) or Mr. James, he must be Mr. Ross or Mr. Haines. The assumption that the president is Mr. Ross leads to the instructor being Mr. Jones which makes (2) and (6) contradictory. Therefore the president is Mr. Haines. If the instructor cannot be Mr. Jones (see above), Mr. Haines, or Mr. James, he must be Mr. Ross. The janitor is therefore Mr. Jones.

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
State Teachers College, Oneonta, New York

THE RULE OF SIGNS IN MULTIPLICATION

THE RECENT discussion* of the rules for determining the sign of the product of two signed numbers suggested a study of the ways in which this question has been treated in algebras in the past. A problem of this type does not merit an exhaustive investigation. The question is how have various authors gone about the task of making this rule reasonable to the reader. The volumes chosen for the study were simply those that happened to be at hand: several of them were very influential; others were perhaps a bit obscure.

In some, a simple enunciation of the rules is all that is given. An example of this occurs in Sir Isaac Newton's *Universal Arithmeticke*, the quotation being from the second edition in English (London 1728).

> Simple Algebraick Terms are multiply'd by multiplying the Numbers into the Numbers, and the Species into the Species, and by making the Product Affirmative, if both the Factors are Affirmative, or both Negative; and Negative if otherwise.

François Legendre's *Arithmétique en sa Perfection* (first printed in 1656 and reprinted in many editions the last being in 1812) devoted a few pages to a summary of algebra in which the rules for the signs of the product of two quantities are called "maximes"

> Quand on multiplie P par P, il vient plus.
> Multipliant M par M, il vient P.
> Multipliant M par P ou P par M, le produit est toujours M.

Edward Hatton is more explicit in his work *An Intire System of Arithmetic* (London, 1721, 3rd edition, 1753)

> The Reason of the General Rule for Multiplication:
>
> 1st—Multiplication being nothing but the Work of many additions; it follows that $-ab$ being added to $-ab$, is $-2ab$ which is the same as multiplying $-ab$ by 2; and so $-gh-2gh-3gh$ added is equal to $-6gh$ which shows that the Negative Quantity $-gh$ by the Affirmative 6 must give the Negative $-6gh$ and the like Reason holds for any other Negative by a positive Quantity.
>
> 2nd—And as in *Logic*, two Negatives make an Affirmative; so in *Algebra* one Negative Number or Quantity multiplied by another Negative produceth an Affirmative; For the Sign − being directly contrary to +, must make a Product of a direct contrary Nature when the Multiplier is −, then (as above) when it is +; as subtracting a Negative, is adding really so much as we seem to subtract, as -2 from 4 is 6.

John Ward's *Young Mathematician's Guide* (London, 1707; 1758 ed.) goes into more detail. The author announces the rules and then says

> That + into + will produce + in the Product, is evident from *Multiplication* in Common *Arithmetick:* viz. +5 into +7 will give +35 &c. But that + into −, or − into + should produce the Sign − and that − into − should produce the Sign +, . . . may perhaps seem somewhat hard to be conceived; and requires a Demonstration.

The demonstration uses the postulates that equals multiplied by equals yield equals, and the same for addition and subtraction. One of his illustrations will suffice. To prove that $-7b$ into $+3f$ equals $-21bf$, he reasons as follows:

suppose	$4a-7b=0$
then	$4a=7b$
but	$+3f=+3f$
multiplying	$12af=21bf$
or	$12af-21bf=0.$

Consequently + into − or − into + produces −. That is, the product of $-7b$ by $+3f$ was found by first finding the product of $7b$ by $3f$.

Bonnycastle's *Introduction to Algebra*

* See Van Engen, H., *On Understanding Mathematical Methods*, THE MATHEMATICS TEACHER, December, 1950, pp. 397–444.

with Notes and Observations had a number of editions in the United States, the first being Philadelphia, 1806. In this text, the explanation of the rules is given in a footnote. The author establishes the product of two positive quantities by assuming that multiplying by $+b$ means taking the multiplicand b times. Then, the other cases are explained as follows:

2nd.—If two quantities are to be multiplied together, the result will be exactly the same in whatever order they are placed; for a times b is the same as b times a: and, therefore, when $-a$ is to be multiplied by $+b$, or $+b$ by $-a$, this is the same as taking $-a$ as many times as there are units in $+b$; and since the sum of any number of negative terms is negative; it is evident that $(-a)\times(+b)$ or $(+b)\times(-a)=-ab$.

3d.—When $-a$ is to be multiplied by $-b$: Here $a-a=0$, therefore $(a-a)\times-b$ is also $=0$ because 0 multiplied by any quantity produces 0; and since the first term of the product, or $a\times(-b)$ is, by case 2, $-ab$, the last term or $(-a)\times(-b)$ must be $+ab$, in order to make the sum $(-ab+ab)=0$ consequently $(-a)\times(-b)=+ab$.

The third New York edition was "revised, corrected, and enlarged . . . by James Ryan." It appeared in 1825. The note on the signs in multiplication differs from that of the Philadelphia edition of 1806. Its explanation is:

The above rule for the signs may be proved thus: if B, b be any two quantities, of which B is the greater, and $B-b$ is to be multiplied by a, it is plain that the product, in this case, must be less than aB, because $B-b$ is less than B; and consequently, when each of the terms of the former are multiplied by a, as above, the result will be

$$(B-b)\times a=aB-ab.$$

For if it were $aB+ab$, the product would be greater than aB, which is absurd.

Also, if B be greater than b, and A greater than a, and it is required to multiply $B-b$ by $A-a$, the result will be

$$(B-b)\times(A-a)=AB-aB-bA+ab.$$

For the product of $B-b$ by A is $A(B-b)$ or $AB-Ab$ and that of $B-b$ by $-a$ which is to be taken from the former, is $-a(B-b)$, as has been already shown; whence $B-b$ being less than B, it is evident that the part which is to be taken away must be less than aB; and consequently since the first part of this product is $-aB$, the second part must be $+ab$; for if it were $-ab$, a greater part than aB would be taken from $A(B-b)$ which is absurd.

The introductions to each of these editions speak of the "moderns" to whom "we are principally indebted for the most curious refinements of the art (algebra) and its great and extensive usefulness in every abstruse and difficult inquiry." Each edition mentions Newton, Maclaurin, Saunderson, Simpson, and Emerson. The edition of 1825 inserts the names of Clairaut, Euler, Lagrange, and Lacroix also and each edition then concludes with the statement that the men cited "have particularly excelled in this respect; and it is to their works that I would refer the young student as patterns of elegance and perfection." There may be no significance in the reference to Lacroix (1765–1843), but Lacroix was a well-known teacher and textbook writer and was successively at the École Normale, the École Polytechnique, and the Collège de France. His *Algebra* was translated "for the use of the students of the University at Cambridge, New England" by John Farrar and was published as a part of the first volume of *Cambridge Mathematics*. The quotation that follows is from the Fourth Edition (Boston, 1833).

When the multiplicand contains parts to be subtracted, the products of these parts by the multiplier must be taken from the others, or in other words, have the sign $-$ prefixed to them, for example:

the product of	$a-b$
multiplied by	c
is	$ac-bc$;

for each time that we take the entire quantity a, which was to have been diminished by b before the multiplication, we take the quantity b too much; the product ac therefore, in which the whole of a is taken as many times as is denoted by the number c, exceeds the product sought by the quantity b, taken as many times as is denoted by the number c, that is by the product bc; we ought then to subtract bc from ac which gives as above,

$$ac-bc.$$

A little more explanation follows and then there is the conclusion:

When the multiplier has the sign $+$, the product has the same sign as the corresponding part of the multiplicand.

The author next discusses the product of

$a-b$ by $c-d$, which has the product

$$ac-bc$$
$$-ad+bd$$

for the product of the multiplicand by the first term of the multiplier, will be by the last example $ac-bc$; but by taking the whole of c for the multiplier instead of c diminished by d, we take the whole of $a-b$ so many times too much as is denoted by the number $-d$; so the product $ac-bc$ exceeds that sought by the product of $a-b$ by d. Now this last is, by what has been said, $ad-bd$, and in order to subtract it from the first it is necessary to change the signs. We have then $ac-bc-ad+bd$ for the result required . . . We conclude that . . . each particular product must have the same sign, as the corresponding part of the multiplicand, when the multiplier has the sign $+$, and the contrary sign when the individual multiplier has the sign $-$.

Warren Colburn's *Introduction to Algebra upon the Inductive Method of Instruction* (Boston, 1836) begins by illustrations,—A is worth a number a of dollars, B is not worth so much as A by a number b of dollars, and C is worth c times as much as B, how much is C worth? If a is greater than b, then $c(a-b)$ is positive, but if b is greater than a, then $ac-bc$ is negative.

Another example follows:

A gentleman owned a number of farms, and each farm was worth a number c of dollars, which was his whole property. He hired money and fitted out a number b of vessels and each vessel was worth as much as one of his farms. All the vessels were lost at sea. How much was he then worth? . . . Now if the number of farms exceeded the number of vessels, he still had some property, but if the number of vessels exceeded the number of farms, the quantity $ac-bc$ is negative and he owed more than he could pay. Hence if a positive quantity be multiplied by a negative the product will be negative.

The author then goes to the problem of $a-b$ multiplied by $c-d$. He shows that the product may be put in the form

$$(a-b)c+(b-a)d.$$

Let it be remembered that $a-b$ has the same numerical value as $b-a$, they differ only in sign. Suppose $a-b=-m$, then by changing all the signs $b-a=+m$. Hence

$$(a-b)c+(b-a)d=-cm+dm=m(d-c).$$

Now if d is greater than c (which is the case when $c-d$ is negative), the quantity $m(d-c)$ is positive.

Suppose both $a-b$ and $c-d$ to be negative, as before; then $b-a$ and $d-c$ will both be positive, and their product will be positive. (The multiplication is then performed). This product is precisely the same as that produced by multiplying $a-b$ by $c-d$. Therefore if two negative quantities be multiplied together, the product will be the same as that of two positive quantities of the same numerical value, and will have the positive sign.

It should be remembered that these examples are not by any means a complete collection, but they have the merit of showing a number of different approaches to the topics from Sir Isaac Newton's enunciation of the rules to the detail of the Lacroix presentation.

LETTERS

Statistics in Secondary School Mathematics—When?

Professor Burr, in his article in the January 1951 issue of THE MATHEMATICS TEACHER, recognizes that "although statistical background is important to the student both for his vocation and in his function as a citizen in a democracy, statistics is unfortunately either absent or forms only a minute part of many curricula."

The question of including statistical ideas in secondary school mathematics has been of concern to writers for at least thirty years. T. Kelley in an article published in *School and Society* in February 1920 writes "Statistics can be used for the making of a fuller knowledge of the past and present. It has values which are basic in the life of a democracy, recognizing that nature provides distributions, not types without variability, and that the problem of a democracy is to utilize the talents, attitudes and training of the individuals of a distribution for the sake of raising the mean of the self-same distribution to higher levels of capacity, attitude and training, etc." Concerning the ability of secondary school pupils to cope with statistical concepts, he adds, "The basic ideas in statistics are entirely comprehensible to the high-school level in intelligence." He also adds, "What better time than the high school age; what better teachers than those trained in mathematics. . . . ?"

In 1933 a course in statistics was given at James Monroe High School in Brooklyn, New York. This is described by E. Hausle in an arti-

(*Continued on page 266*)

TOPICS OF INTEREST TO MATHEMATICS TEACHERS

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Meaning, Rigor and Discipline in Secondary Mathematics

FOR a considerable number of years now we have been greatly concerned about the slow learners in mathematics. Until the relatively recent advent of "second track" mathematics, the standard courses in algebra and geometry were gradually watered down until they no longer offered a challenge even to moderately able pupils. Nor has the somewhat checkered career of "general mathematics" helped matters very much. Isn't it about time that we gave a little thought to the more able pupils—there are some still, of course—and stop deceiving them as to the nature of mathematics? The appearance in the literature during the last two or three years of several excellent papers dealing with a more rigorous treatment of elementary mathematics would seem to suggest that the time is ripe for some changes in this direction.

We do not wish to revive the traditional courses of a quarter of a century ago, of course. Yet it seems feasible to introduce sufficient rigor and substance into mathematics for our better pupils so that they will have something to chew on intellectually.

Consider elementary algebra. How do we "rationalize" operations with signed numbers? Do we emphasize sufficiently the meaning of generalization? Of induction? How clear do we make the concept of operations? the laws governing operations? the extension of concepts, such as fractions, irrationals, and exponents? Do we really teach algebra as generalized arithmetic? There is ample justification for teaching *demonstrative algebra* instead of a more or less arbitrary algebra in which the whys and wherefores remain unexplored. Why not let the pupil realize the role played by postulates and undefined terms in algebra, even as we purport to do in teaching geometry? If it be objected that he is not yet mature enough for this in the ninth grade, which may well be true, then let us defer algebra until *after* demonstrative geometry. After all, what is so sacrosanct about teaching algebra before geometry?

And even here, in the teaching of demonstrative geometry, how successful are we? Do our pupils really understand the meaning and significance of postulational thinking? Do they understand the nature of a hypothetico-deductive system? I strongly doubt it. Most of them, to paraphrase E. T. Bell, have the experience of being black-jacked into memorizing a string of theorems without the faintest notion as to the reason for their sequence, and without the slightest inkling that we can get out of a postulational system only what we put into it—though to be sure, most of the implications are not at once apparent.

In short, our plea is that we put some teeth into high school mathematics. Let our pupils learn something about the logical notions involved; the difference between inductive and deductive thinking; the abstract nature of numbers and other symbols; the power of generalizations; the meaning of necessary and sufficient conditions; the unique significance of postulates, primitives, definitions, and implications; the nature of indirect reasoning.

It is a challenge well worth while to pupil and teacher alike.

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MATHEMATICAL MISCELLANEA

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22. Euclidean Constructions with Well-Defined Intersections

Suppose we consider a point of intersection of two loci as *ill-defined* if the two loci intersect at the point in an angle less than some given small angle θ, say $\theta = 5°$, and that otherwise the point of intersection will be considered as *well-defined*. The question arises: Can any given euclidean construction be accomplished by using only well-defined intersections? Although we are posing, and shall therefore treat, this question as a purely academic one, it might be that it has a practical aspect in connection with drafting. Incomplete results, which in part contain an invalidating oversight, have been given by Hilda Hudson.[3] It is the purpose of this note to remedy these defects and to answer the question in the affirmative. It will be convenient to consider four auxiliary constructions.

CONSTRUCTION 1. *To find, by well-defined intersections, the point of intersection of two lines cutting at an angle less than* θ.

Let the two lines be m and n (see figure 1), intersecting in the ill-defined point O. At approximately equal distances from O erect perpendiculars to m cutting m at A and B and n at C and D respectively. By

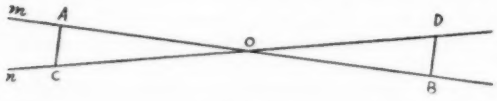

FIG. 1

similar triangles we have $AO:OB = AC:DB$, whence $AO:AB = AC:(AC+DB)$. Since AB, AC, and $AC+DB$ are well-defined distances, we may, by the familiar method of elementary geometry and by using only well-defined intersections, construct the length AO. The point O is then easily located.

CONSTRUCTION 2. *To find, by well-defined intersections, the points of intersection of a line and a circle cutting at an angle less than* s.

Let l be the line and c the circle (see figure 2), intersecting in the ill-defined points M and N. We may, using well-defined intersections, draw the perpendicular OP to the line l from the center O of circle c. Let OP cut the arc MN in the point Q, let the radius of c be chosen as unit length, and designate angle POM by α. Then $OP = \cos\alpha$. By the familiar method of elementary geometry and by using only well-defined intersections, we may construct a length of measure $\cos^2\alpha$, as the third proportional to 1 and $\cos\alpha$. We may then readily obtain a length of measure $\cos 2\alpha = 2\cos^2\alpha - 1$. The process may be continued until we obtain the length $OP' = \cos(2^n\alpha)$, where $2^n\alpha$ is greater than θ. We may thus construct a central angle $QOM' = 2^n\alpha$. By a succession of n bisections, which can be accomplished by well-

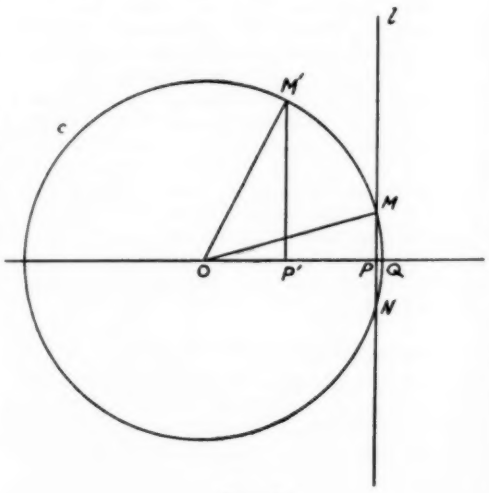

FIG. 2

[3] Hilda Hudson, *Ruler and Compasses* (1916), pp. 109–112.

defined intersections, performed on arc QM', we find the point M. Point N is then easily found.

CONSTRUCTION 3. *To find, by well-defined intersections, the radical axis of two circles cutting at an angle less than θ.*

Let c and d be the two circles. Draw any circle e cutting circles c and d in well-defined points. Since the three radical axes of three circles taken in pairs are concurrent,[4] the common chord of c and e and that of d and e intersect on the radical axis of c and d. To find this point of intersection we may have to use construction 1. Another point on the radical axis of c and d can be similarly found, and the radical axis drawn.

CONSTRUCTION 4. *To find, by well-defined intersections, the point of intersection of two circles cutting at an angle less than θ.*

By construction 3 draw the radical axis of the two circles. The problem is now reduced to construction 2.

It is now evident, in view of the above auxiliary constructions, that we are able to state the

THEOREM. *Any euclidean construction may be accomplished by using only well-defined intersections.*

It is possible to prove more than the above theorem. In addition to the given small angle θ, let us be given a small linear distance d. Suppose we define a euclidean construction to be *well-defined* if it utilizes only well-defined intersections, straight lines determined by pairs of points which are a greater distance than d apart, and circles with radii greater than d. Then it can be shown that *any euclidean construction can be accomplished by a well-defined one*. The treatment of this more general theorem is too lengthy to be given here.

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[4] Cf. P. H. Daus, *College Geometry* (1941), p. 78.

23. The Product of Two Numbers Each Ending in 5

Let us consider the product of two numbers ending in five, each number having two digits. Let x be the ten's digit in the first and larger number, and let y be the ten's digit of the second number. Then

$$10x+5 = \text{the first number,}$$

and $10y+5 = \text{the second number.}$

$$(10x+5)(10y+5) = P,$$

the product of the two numbers.

or

$$100xy+50x+50y+25 = P.$$

or

$$50xy+50x+50xy+50y+25 = P.$$

$$100\,\frac{[(xy+x)+(xy+y)]}{2}+25 = P.$$

or

$$\frac{100[x(y+1)+y(x+1)]}{2}+25 = P.$$

The expression

$$(a) \qquad \frac{x(y+1)+y(x+1)}{2}$$

is the arithmetic mean of the products of the ten's digit of one number and one plus the ten's digit of the other.

When

$$x = y, \quad \frac{x(y+1)+y(x+1)}{2}$$

becomes

$$(b) \qquad x(y+1) \quad \text{or} \quad y(x+1).$$

When

$$x-y = 2, \quad \frac{x(y+1)+y(x+1)}{2}$$

becomes

$$(c) \qquad \frac{2xy+x+y}{2} = \frac{2y(2+y)+2+2y}{2}$$

$$= y(2+y)+1+y$$

which shows that an integral arithmetic mean is obtainable in this case. In general, when $x-y$ is any positive *even* number, $2xy+x+y$ is divisible by 2 and an integral arithmetic mean is obtainable. When

$$x-y=1,\ \frac{2xy+x+y}{2}=\frac{2y(1+y)+1+2y}{2}$$

and the mean is

(*d*) $\qquad y(1+y)+y+1/2$

which differs from an integral mean by .5. In general, when $x-y=k$ is any positive *odd* number the mean (*a*) differs from an integer by an amount equal to one half of $x-y$ and is given by

(*e*) $\quad y(k+y)+y+k/2$, or $xy+y+k/2$.

This argument provides a short cut for calculating the products of pairs of integers ending in 5. For example:

(*f*) $\quad 35\times35=100(3)(3+1)+25=1225.$

See (*b*) above.

(*g*) $\quad 35\times15=100[3(2)+4(1)]/2+25$
$\qquad =525.$

See (*a*) above. Or,

$\qquad 35\times15=100[1(3)+1+1]+25=525.$

See (*c*) above.

(*h*) $\quad 85\times75=100[7(8)+7+.5]+25$
$\qquad =6375.$

See (*d*) above.

(*i*) $\quad 85\times55=100[5(8)+5+1.5]+25$
$\qquad =4675.$

See (*e*) above.

(*j*) $\quad 95\times25=100[2(9)+2+3.5]+25$
$\qquad =2375.$

See (*e*) above.

Note the special rule for squaring a number ending in 5 as in 35×35 above. The result may be obtained by writing down 25 and then prefixing to it the product of the units digit (3) with one plus the units digit (4).

The most general rule (*e*) may also be expressed in words as: Multiply by 100 the sum of the product of the ten's digits plus the smaller ten's digit plus one-half the difference of the ten's digits, then add 25.

The above arguments using two digit numbers may be appropriately extended to numbers of more than two digits.

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24. The Generation and Use of Heronian Triangles

Problems which illustrate and apply numerical properties of geometric figures, as studied in geometry and in trigonometry, frequently lead to complexities in computation because of irrationality. In order to place emphasis upon the principles and processes being applied it is well to lighten the burden of computation by arranging that the quantities involved are rational, or even integral. It has already been noted in these columns that Pythagorean numbers furnish a fund of raw material for constructing such problems. [The Mathematics Teacher, Vol. XLIII (April, 1950), p. 102; (Nov., 1950), p. 352.] Another source of rational data is the rational triangle—an oblique triangle whose area and lengths of sides are rational. Many associated quantities—altitudes, segments on the sides formed by the feet of the altitudes, circumradius, inradius, exradii, trigonometric functions of the angles—are rational. Consequently such a triangle may be taken as the basis for a variety of these problems.

An oblique triangle with integral sides and area need not have an integral altitude. Witness the triangle with sides 100, 143, 153, having area 6930; also the triangle with sides 10, 35, 39 and area 168. Neither triangle has an integral altitude. By "Heronian triangle," however, we mean a triangle that does have an integral altitude as well as integral sides and areas.

It is easy to generate Heronian triangles by compounding two Pythagorean triangles. Take, for example, the Pythagorean primitives 5, 12, 13 and 8, 15, 17. Since 24 is the L. C. M. of 12 and 8, these triangles may be suitably enlarged so that each has a leg 24. Then, placing the triangles with the equal legs in coincidence, the Heronian triangles 26, 51, 55 and 26, 35, 51 result, depending on whether or not the triangles are placed so as to overlap. Tables of Pythagorean numbers may be constructed and from them tables of Heronian triangles. There would be eight entries in the latter table for each pair in the former as shown:

Pythagorean Triangles

Hyp.	*Legs*	
5	4	3
13	12	5

Heronian Triangles

	Sides		*Perimeter*	*Area*
13	15	14	42	84
13	15	4	32	24
39	25	16	80	120
39	25	56	120	420
13	20	21	54	126
13	20	11	44	66
52	25	63	140	630
52	25	33	110	330

A triangle, 100, 143, 153, with integral area but no integral altitude may be formed as follows. Take the right triangles 65, 63, 16 and 85, 77, 36, whose hypotenuses have the common factor 5. Compound them into a Heronian by matching the legs 63 and 77, taking their L. C. M., $7 \cdot 9 \cdot 11$, to be the altitude. The oblique sides are $11 \cdot 65$ and $9 \cdot 85$. The base is $11 \cdot 16 + 9 \cdot 36 = 500$, also having 5 as a factor. The desired triangle results when the common factor is removed.

Various properties and some special classes of Heronian triangles can be investigated by elementary methods. These furnish interesting topics for club meetings or other activity periods. As an example, consider Heronian triangles whose sides are consecutive integers, say $2m-1$, $2m$, $2m+1$. It turns out that the general form of $2m$ is given by

$$2m = (2+\sqrt{3})^n + (2-\sqrt{3})^n, \; n \text{ an integer.}$$

Successive values of n give rise to the following series of triangles:

Sides	*Area*
1, 2, 3	0
3, 4, 5	6
13, 14, 15	84
51, 52, 53	1,170
193, 194, 195	16,296
723, 724, 725	226,974
etc.	

Readers who wish to pursue the subject further may consult the following:

Carmichael, R. D., *Diophantine Analysis.* New York: John Wiley & Sons Inc., 1915, pp. 8–13.

Alliston, N., *Mathematical Snack Bar.* Cambridge (England): W. Heffer & Sons, Ltd., 1936. pp. 116–125.

Dickson, L. E., "Rational Triangles and Quadrilaterals." *American Mathematical Monthly,* vol. 28 (June-July, 1921), p. 244.

Cheney, W. F., Jr., "Heronian Triangles." *American Mathematical Monthly,* vol. 36 (Jan., 1929), p. 22.

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25. The Improper Use of "=," and a "Pet Peeve"

In response to an invitation issued earlier in this column Helen G. Crozier, Head of the Mathematics Department of Fortuna Union High School, Fortuna, California, writes:

"My pet peeve is the misuse of the equality sign. In substituting in formulas, pupils often do this sort of thing:

$$A = 1/2bh, \qquad b = 8, \qquad h = 12$$

$$A = 1/2 \cdot 8 = 4 \cdot 12 = 48.$$

To avoid this I have them substitute all values at once, writing the numerical values right under the letters, and writing only one equals sign on a line, thus:

$A = 1/2bh, \quad b = 8, \quad h = 12$

$A = 1/2 \cdot 8 \cdot 12$

$A = 4 \cdot 12$

$A = 48.$

Also in checking algebraic equations, I have students use a '?' above the equal sign until the last step when an identity is obtained, then use a check mark ($\surd$) over the equal sign. In this way, I try to have students realize and recognize what 'equals' really means."

The editor of "Miscellanea" cannot resist adding in this same vein his "peeve" against the use of "=" in trigonometric identities which are to be proved. This unfortunate use of a symbol is a major reason for our trouble with students who in these proofs operate on both sides as if we had an equation. The editor uses the question mark device with much verbal emphasis and insistence upon it to indicate the supposedly unsettled status of these trigonometric problems. Incidentally, should we not include in the problems for this section of trigonometry some statements which are not true? For the students to be able to discover and explain the conditions for equality to exist would be far more valuable (and harder) than for them to "prove the identity" when told to do so and to "solve the equation" when so instructed.

Letters

(*Continued from page 258*)

cle appearing in the January 1937 edition of THE MATHEMATICS TEACHER. The experiment was conducted for the purpose of answering the question, "Can a course in statistics be given to secondary school pupils?" All the pupils enrolled in the course had completed Intermediate Algebra and were taught not only statistical concepts but operational statistics as well. According to Hausle, "comments of the pupils who took the course indicate that they derived a real benefit and enjoyment from it because they could see a meaningful application of their mathematics." As for Hausle's personal opinion, she feels that such a course should become a unit in every high school wherever possible.

In 1946, a Committee on Collecting and Analyzing data at Duke University (see MATHEMATICS TEACHER, April 1946) came out strongly in favor of including statistics in high school. "The study of statistics offers unlimited opportunities for applications of mathematics in fields where social problems exist; business, industry, government, agriculture, medicine, psychology, sociology and education are enormously rich in problems, data and opportunities. Statistics enters into surveys, analyses, norms, testing, etc. The committee recommended: (1) that at least some statistical training be given to all high school students, and (2) that opportunities for broadening experiences with statistical points of view and techniques be made available for those high school students who wish them.

The three references cited represent the general feelings of most writers on the subject of statistics in the secondary school. A survey by the writer of all the literature that could be found on the subject reveals almost unanimous agreement on the following points:

1. There is much of value to be obtained through the study of statistics at the secondary school level and that mathematics is the subject best equipped for this purpose.

2. Pupils of secondary school mathematics are capable of mastering many statistical ideas.

3. The study of descriptive statistics with emphasis on concepts is of greater value than the study of statistics from an operational point of view.

Having outlined briefly the firm position of writers in the past on the matter of statistics as a secondary school tool, we now feel justified in asking, "Why the delay during all these years in introducing statistics in the secondary school mathematics curriculum?" The writer does not profess to know the answer. One possibility is that the value of the study of statistics has never, as far as is known, actually been put to test by experimentation, to warrant its being placed in the mathematics curriculum at the expense of some of the traditional topics in mathematics. It should be remembered that the experiment conducted at James Monroe High School in 1933, though reflecting favorably upon the study of statistics, was restricted to students comparatively advanced in secondary school mathematics. An investigation into the content and placement of statistical ideas for all students at the secondary school level has been long overdue and would represent a significant educational contribution. It could help to make curriculum committees more aware of the value of incorporating more statistical ideas in the mathematics curriculum.

As a parting word, we wish to mention that a recent publication by the Board of Education of the City of New York, (*General Mathematics for*

(*Continued on page 270*)

INSTITUTES, WORKSHOPS, CONFERENCES

Duke University, Durham, North Carolina, Institute for Teachers of Mathematics, Eleventh Annual Session, August 6–17, 1951.

Theme: "Mathematics at Work." The following persons are scheduled to lecture to the Institute: Ralph M. Berry, Mathematician, Coast and Geodetic Survey; Paul Boder, Scientist, American Optical Company; C. G. Brenecke, Chairman, Electrical Engineering Department, North Carolina State College; J. W. Cell, Professor of Mathematics, North Carolina State College; H. A. Fisher, Chairman, Department of Mathematics, North Carolina State College; J. J. Gergen, Chairman, Department of Mathematics, Duke University; Bain Griffith, Designing Engineer, Chevrolet Company; Capt. R. Hatcher, Bureau of Aeronautics, U. S. Navy; Philip O. Johnson, Designing Engineer, Fisher Body Company; Phillip S. Jones, Department of Mathematics, University of Michigan; E. S. Lee, General Electric Company; W. D. Merrifield, Industrial Education, Chrysler Corporation; Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin; W. W. Rankin, Department of Mathematics, Duke University; Robert Schilling, Engineer, General Motors Research Laboratory; H. A. Shannon, North Carolina State Education Department; Col. R. B. White, President, Baltimore and Ohio Railroad; W. M. Whyburn, Chairman, Department of Mathematics, University of North Carolina; and C. A. Wright, Statistician, Standard Oil Company.

The following study groups will meet daily:

I. Aids in the Study of Algebra (Jr. and Sr. H.S.). Leader: Frances Johnson, Oneonta High School, Oneonta, New York.

II. Mobilization and the Mathematics Curriculum. Leader: Veryl Schult, Director of Mathematics, Washington, D. C.

III. Algebra (H.S. and Col.). Leader: J. J. Gergen, Chairman, Department of Mathematics, Duke University.

IV. The Slow Student in Mathematics (Jr. and Sr. H.S.). Leader: Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin.

V. General Mathematics (Jr. and Sr. H.S.). Leader: Nanette R. Blackiston, Supervisor of Mathematics, Baltimore, Maryland.

VI. Trigonometry and Analytic Geometry (H.S. and Col.). Leader: J. W. Cell, Professor of Mathematics, North Carolina State College.

VII. History—Philosophy—Psychology of Mathematics (Jr. and Sr. H.S. and Col.). Leader: Phillip S. Jones, Assistant Professor of Mathematics, University of Michigan.

VIII. Uses of Mathematical Instruments (Jr. and Sr. H.S.).

IX. Laboratory Mathematics (Jr. and Sr. H.S.). Leaders: Ida Mae Bernhard, Laboratory School, San Marcos, Texas; and Allene Archer, Jefferson High School, Richmond, Virginia.

X. Laboratory Mathematics (H.S. and Col.). Leaders: Amelia Richardson, McKeesport High School, McKeesport, Pa., and Phillip S. Jones, University of Michigan.

On the social side, there will be one banquet, three dinner meetings, one reception, one tea, one organ recital, one carillon recital, and a watermelon party.

For further information write: Professor W. W. Rankin, Director, Duke Mathematics Institute, Duke University, Durham, North Carolina.

University of Houston, Houston, Texas. First Mathematics Institute. July 30 through August 2, 1951. This institute is being held under the joint sponsorship of the College of Education, the Department of Mathematics, and the College of Business Administration. Each day's program will begin with a general session. On Monday, July 30, Dr. M. M. Slotnick, Chief Mathematician of the Humble Oil and Refining Company, will address the first session. Dr. W. A. Brownell, Dean of the School of Education at the University of California will be in charge of sections dealing with elementary mathematics, Dr. E. H. C. Hildebrandt of Northwestern University will be in charge of sections dealing with the problems of high school teachers and Dr. E. T. Bell, Professor of Mathematics at California Institute of Technology will direct the college section. Each of these men will address a general session also. Afternoons will be devoted to small study groups for teachers at the various levels of teaching.

All meetings will be held in air-conditioned rooms. Rooms will be available in air-conditioned dormitories at $1.50 per night. The fee for the entire period of the Institute will be $10.00 and a certificate of attendance will be issued to those participating.

Additional information will be sent to those writing to Dr. Esther F. Gibney, Box 573, University of Houston, 3801 Cullen, Houston, Texas.

Indiana Workshop for Mathematics Teachers. Indiana University, in cooperation with Purdue University, will hold its Fourth Annual Workshop for Mathematics Teachers on the Bloomington campus beginning **Monday, June 25** and closing **Saturday, July 7.** The Workshop will include opportunity for discussion of your particular problems, a Visual Aids Laboratory where you may try your hand at making things, speakers from industry and business, as well as specialists in the field of mathematics. The program should be available around May 15. Write to Philip Peak, Indiana University, Bloomington, Indiana.

Northwestern University Workshop in Multi-Sensory Aids in the Teaching of Mathematics, August 6–August 24, 1951. This will be a discussion and laboratory course on instructional and learning aids which may be used to

(*Continued on page 277*)

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by JOHN R. MAYOR *and* PAUL M. WEISEL
The University of Wisconsin, Madison, Wisconsin

A TABLE summary of the 15 replies to the questions on Mathematics Enrollments, published in this section in the November, 1950, number of THE MATHEMATICS TEACHER is given below. Since the small number of replies does not provide sufficient information on which to base general conclusions on what is going on in our schools, this first set of questions is repeated with the hope that additional replies can be had. The questions are:

A. Mathematics Enrollments
1. How many years of sequential mathematics (in grades 9–12) is it possible for a graduate of your school to take?
2. What is the mathematics requirement for graduation from your high school?
3. How long has this requirement been in effect?
4. What per cent of your graduates of 1950 completed
   a. one year of mathematics in grades 9–12?
   b. two years of mathematics in grades 9–12?
   c. three years of mathematics in grades 9–12?
   d. four years of mathematics in grades 9–12?
5. What per cent of your graduates of 1950 entered college this fall?

Please send the answers to these questions for your school to John R. Mayor, North Hall, Madison 6, Wisconsin by May 15, 1951. A summary of replies will be published in the October issue. We believe that this section can be most effective if replies are sent in promptly and promptly reported.

Readers of THE MATHEMATICS TEACHER are also invited to submit questions for publication in future issues.

In replying please give your name, the name and locality of your school, and your high school enrollment. Also, please indicate the organization of your school such as 4-year high school, grades 9–12; senior high school, grades 10–12; junior high school, grades 7–9; etc. The questions may be answered by number without use of a particular form or repetition of the questions.

In reports no reference will be made to names of teachers or schools.

The 15 replies to the questions on page one, received by December 15, came from 13 different states: Illinois, Iowa, Massachusetts, Michigan, Minnesota, Nebraska, New Hampshire, North Carolina, Oregon, Pennsylvania, Rhode Island, Texas, and Wisconsin. Twelve of the reports are from public schools and three from private schools.

While no conclusions can be drawn from this table it will be interesting for schools to compare their own registrations with that of other schools of similar organization and size. To facilitate one kind of comparison the schools have been listed in order of per cent of graduates of 1950 entering college.

Another interesting comparison for these schools is had by observing the effect, if any, of a requirement of one or two years of mathematics on the enrollment in the third and fourth year courses.

The fact that all except one of the schools reporting offered at least four years of sequential mathematics would, of course, indicate one characteristic of this group of schools which unfortunately is not true of all schools even as large as these. Some might suppose that those schools which had the best record in terms of per cent of students, who continue the study of mathematics through and beyond the second year, would be most apt to report, but this should not be so since names and teachers are not to be revealed. For similar questions, the information in this table is in reasonable agreement with that reported by Schorling

(THE MATHEMATICS TEACHER, April, 1948). In the Schorling survey nearly 300 replies were received, but a much longer period was allowed for collection of data.

MATHEMATICS ENROLLMENTS IN 15 SECONDARY SCHOOLS

(The numbers at the tops of the columns are the numbers of the questions above; the answers to questions 1, 2, and 3 are in years; the answers to questions 4a, 4b, 4c, 4d, and 5 are in per cents.)

School	Grade Organization	Enrollment	1	2	3	4a	4b	4c	4d	5
A	9–12	740	4	3	50	100	100	100	80	100
B	9–12	112	4	2	—	100	100	10	0	100
C	7–12	325	4	1	20	100	86	48	20	73
D	Senior	—	4	1	10	100	90	20	10	70
E	9–12	400	6	2	15	100	100	40	16	54
F	7–9, 10–12	450	4	2	10	100	100	40	20	45
G	Senior	—	4	0	10	100	36	17	14	40
H	9–12	610	4	2	5	100	89	30	9	30
I	9–12	—	3½	2	10	100	100	27	9	30
J	9–12	900	4	—	—	100	100	30	10	25
K	10–12	786	4	2	20	100	100	20	10	25
L	9–12	1297	4	0	—	79	62	34	11	24
M	9–12	485	4	2	20	100	97	24	6	19
N	9–12	325	4	1	—	86	49	10	—	17
O	7–12	245	4	1	10	100	64	44	0	16

MATHEMATICAL RECREATIONS

Edited by AARON BAKST

School of Education, New York University, New York 3, N. Y.

THE Christmas greeting extended by this department to the readers of THE MATHEMATICS TEACHER elicited several interesting responses. MR. FRANK J. HARDMAN of the Southern Junior-Senior High School, Baltimore, Maryland submitted the following greetings:

$$\frac{(90°-C_1)(90°-C_2)(90°-C_3)\cdots(90°-C_n)}{2u}$$

which is deciphered as: COMPLEMENTS OF THE *C*'S TO YOU.

$$\{[(X+A)(X-A)M+A^2M]E\}$$

$$\cdot\left\{\left[\frac{\dfrac{R+X}{X}+\dfrac{X}{R-X}}{\dfrac{X}{R-X}}\right]\right.$$

$$-\left[\frac{4AS}{3X[(E+Y)^2-(E-Y)^2]}\right.$$

$$\left.+\frac{4AS}{3E[(Y+X)^2-(Y-X)^2]}\right.$$

$$\left.\left.+\frac{4AS}{3Y[(X+E)^2-(X-E)^2]}\right]\right\}Y.$$

The pupils who will have patience enough to perform all the indicated operations will be rewarded with

$$MER^2Y-ASXM=MERRY\ XMAS.$$

This problem is suitable for an Intermediate Algebra class.

This department wishes to offer its apologies to MISS HAZEL DULING who is responsible for the origin of the problem "A MATHEMATICAL ROMANCE" which was published in the December issue of THE MATHEMATICS TEACHER. MISS MARGARET JOSEPH called the attention of this department to the fact that the idea of this problem originated with MISS DULING.

The solution of this problem is given

below. The missing words are listed in the order of their appearance.

1. major 2. premise 3. axis 4. powers 5. degree 6. slope 7. obliquely 8. erect 9. point 10. survey 11. area 12. base 13. cone 14. symmetry 15. sign 16. inclined 17. isosceles 18. acute 19. triangle 20. general 21. triangle 22. sum 23. plane 24. line 25. height 26. similar 27. regular 28. consecutive (or alternate) 29. functions, 30. complementary 31. chords 32. tangent 33. extreme 34. radical 35. irregular 36. left 37. right 38. correspond 39. distance 40. proposition 41. postulate (or assumption) 42. extended 43. assumed 44. erect 45. quadrangle 46. adjacent 47. space 48. section 49. period 50. acute 51. converse 52. pi 53. space 54. loci 55. inscribed 56. triangle 57. straight 58. exterior 59. revolve 60. sphere 61. premise 62. obtuse

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Geometric recreations are generally difficult to obtain. This department would like to secure problems which would lead to original work by the pupils. For example, here is a problem which might be generalized into a series of additional problems. Given a right triangle ABC with C as the right angle. Draw the altitude CD. The sum of the radii of the circles which are inscribed in the triangles ABD, ACD and BCD is equal to the length of the altitude CD. (This should be proved.)

Suppose we take any triangle ABC (this time not a right triangle). Draw the altitude CD. Determine the sum of the radii of the circles which are inscribed in the triangles ABC, ACD and BCD.

We may generalize this problem further. Suppose that we draw the three altitudes. Let the orthocenter be O. We then have three triangles AOB, AOC and BOC. What will be the relation of the sum of the radii of the circles which are inscribed in these three triangles to the radius of the circle which is inscribed in triangle ABC?

The same situation may be examined if we replace the altitudes by the medians or by the angle bisectors of the triangle.

Solutions by the readers will be welcome and will be published in this department. The greatest advantage which may be derived from the study of geometry is associated with the opportunity of variation of certain given conditions stated in a problem, whether this is an original or a construction. If the pupils are encouraged to vary the conditions, they will eventually develop the tendency to "imagine" and to formulate problems of their own. We often hear so much about "motivation" in instruction. The only "motivation" which is effective is the "self-motivation" by the pupils.

Letters

(*Continued from page 266*)

the 9th Year, Curriculum Bulletin, 1949–50 Series, p. 35) does recommend several statistical ideas for use in connection with a unit on "Graphic Presentation." However many of the topics are listed as optional. In addition, the publication is limited for use with 9th year mathematics pupils "whose interest in mathematics is of a general nature and who will not have the need for preparing for more advanced courses in mathematics."

MARTIN HIRSCH
Junior High 227
Brooklyn, New York

Topics of Interest

I read with interest the article in the February issue of THE MATHEMATICS TEACHER on "Topics of Interest to Mathematics Teachers." But how is one going to secure them to read? I should think you would have a large market for a book comprising all of those articles.

Should you decide to undertake the task, consider this my order for a copy.

BEA BUZZETTI
Bremerton, Washington

AIDS TO TEACHING

Edited by

HENRY W. SYER
School of Education
Boston University
Boston, Massachusetts

and

DONOVAN A. JOHNSON
College of Education
University of Minnesota
Minneapolis, Minnesota

BOOKLETS

B.57—Mathamerica

Markilo Manufacturers, 3633 S. Racine Avenue, Chicago 9, Ill.

Booklet; 4½″×7″; 44 pp; Single copies —$.25.

Description: The booklet is concerned with the simplifications which would result from replacing our decimal number system by one whose base is twelve rather than ten. Perry cites advantages in merchandising, mensuration, and general simplification of operation. Disadvantages of our number system are also mentioned. Solutions are then given to problems in counting and performing the four fundamental operations.

Appraisal: The booklet is primarily useful in stimulating thought concerning our decimal number system. Reference to the duodecimal system serves to bring to the light many concepts which would be missed in our system because they seem so fundamental and unquestionable.

The author develops his thesis well but is almost religiously enthusiastic in recommending adoption of the radix twelve. Operations are developed psychologically —from the simple to the difficult. The vocabulary and level of the concepts developed make the booklet suitable for superior pupils only. It might prove very interesting and enlightening to elaborate upon the idea by means of class discussion of advantages and disadvantages of the duodecimal systems. Bases other than ten and twelve may also be studied.

The blue-covered booklet is printed upon poor paper and the printing is unattractive. The general appearance of the booklet is poor and the staples which serve as a binding are not sufficiently substantial to survive many readings.

The booklet represents a worthy addition to the classroom library regardless of its poor physical qualities. (Reviewed by Bernard Singer, Hyannis, Mass.)

B.58—Streamlined Methods of Computing with Slide Rule and Mathematical Tables

Donald V. Mitchell, 12345 Sand Point Way, Seattle 5, Wash.

Booklet; 5½″×8½″; 80 pp; $1.50.

Description: The booklet, written by Mitchell, is a notebook insert designed for purposes of mathematical practice and review. It considers topics required in structural shop-work. Among the subjects discussed are the art of speedy computation, the use of formulas in determining square and cube roots and solving quadratic equations, use of the slide rule, and computing with logarithms. Facility in the use of tables and slide rule is considered essential in solving problems arising in the shop.

Appraisal: The information is well arranged but the printing is dark, poorly spaced, and difficult to read. The pages and cardboard cover are of good quality.

Material discussed in the booklet is appropriate because it is practical rather than theoretical; students of shop mathematics are more interested in the instrumental value of the subject than in its logical or theoretical implications.

The price is within reason if the texts being used do not include the subjects presented in the booklet. It is worthy of mention that a 47-page book of tables may be purchased as a supplement to the booklet discussed above; it is entitled "Six-Place Tables for Precision Computing" and costs $1.00. (Reviewed by Bernard Singer, Hyannis, Mass.)

CHARTS

C. 24—Dimensions of Natural Objects (71972)

Central Scientific Co., 1700 Irving Park Road, Chicago 13, Ill.

Chart; 28″×45″; $1.00

Description: This chart is a black lithograph reproduction of a mural which was painted for the Hall of Science at the 1933 Century of Progress Exposition held in Chicago. It indicates the relative dimensions of eleven natural phenomena ranging in size from the proton to the galaxy. A simple diagram of each object appears beside the dimension which describes it. The dimensions are so arranged that the decimal points lie in a vertical line; numerical comparisons are thus easily made and relative sizes are almost immediately obvious.

The printing consists of a large number of tiny black dots which fuse, when viewed from a distance, to form the desired diagrams and lettering. Close study is ineffective however, for the dots are then plainly visible.

Appraisal: The chart may be used effectively to introduce logarithms and the scientific exponential notation whose usefulness in expressing extreme values can not be minimized. It may likewise be used to supply interesting data for exercises which require conversion from one set of units to another—from miles to light years, from inches to yards, etc.

The vertical arrangement of the numerical values is excellent for making comparisons but the chart does not have any other exceptional physical qualities. The price seems to be sufficiently reasonable to justify its purchase. (Reviewed by Bernard Singer, Hyannis, Mass.)

EQUIPMENT

E. 38—Spinno

The John C. Winston Company, Philadelphia, Pennsylvania

Game-drill device; Square base 6″×6″; 8 disks 6″ radius; shields and spinner; $1.65 plus postage.

Description: The base of this device is of heavy cardboard and contains a pivot in the center. The eight disks, printed on both sides, contain four sets of practice examples for each of the four fundamental processes: addition, subtraction, multiplication and division; there are 25 combinations on each side. The four sides for each combination are graded from easy to difficult. The answers are printed under each combination but the smaller circular shields fitting over the pivot allow only one answer to be seen, and the probability that it would be the one on which the pointer stopped is small; in fact, by putting on both shields all answers could be covered. A booklet of directions of four pages ($5\frac{1}{2}$″×$5\frac{1}{2}$″) is included with suggestions for drill games.

Appraisal: The manufacturer suggests that the game is suitable for grades 3 and 4 for learning and for grades 5 through 8 for remedial work. This distinction is moot, but the general estimate seems fair. The graded exercises seem well chosen, and cover a sufficient range of material. Zero facts are included right from the beginning. The physical appearance is colorful and sturdy. When a device consists of separate parts there is always the danger that some will be lost or bent, but since all parts can be kept on the center pivot at all times, this danger should be reduced. There seems little reason for supplying these in quantity to the whole class, but a growing collection of the interesting drill-games constantly appearing should certainly include this one.

FILMSTRIPS

FS. 81–89—Decimals and Percentage

Eye Gate House, 330 W. 42nd Street, New York 18, N. Y.

Filmstrip; 35 mm; color; silent; $4.00 each ($22.50 for the set).

Description of FS. 81—Introduction to Decimals (25 frames): The filmstrip begins with pictures showing a few great

advances in mathematics. The familiar "pie graph" is used to present fractions as a way of expressing things that are smaller than one. A problem is then given to show that operations with fractions are sometimes difficult. It is proposed that this be overcome by the use of fractions with 10, 100, or 1000 as a denominator. Two frames are used to present the idea of the decimal point. Following this is the decimal use in finance. The superiority of decimals over common fractions in business is shown by placing price tags on various pieces of athletic equipment. A final picture shows several ways of using decimals.

Appraisal: The approach is good but, aside from the first four frames, most of the material might be illustrated as well on the blackboard. Frames 22–23 are good for showing the difference of writing prices, fractions, and decimal forms.

Description of FS. 82—Decimals and Common Fractions (25 frames): Frames 3-6 show John measuring a bicycle seat, a knife, and a pencil with a meter stick. Next, John takes a paper divided into 100 squares and paints 10 squares—showing the idea of one-tenth. A penny and a dime are shown to be fractional parts of a dollar. Frame 11 presents positional notation after which several problems are given for illustration. Finally, a candy bar is used to demonstrate fractional parts—both common and decimal fractions are given.

Appraisal: There is some question in my mind as to whether or not students of this age are familiar with the metric system. There is hardly a thing shown in this filmstrip that could not be illustrated better by use of the actual object, or by use of the blackboard.

Description of FS. 83—Adding and Subtracting Decimals (25 frames): This filmstrip begins with problems and examples of adding and subtracting decimals. A considerable portion of the film is used to emphasize the importance of "placing the decimal point."

Appraisal: The problem in Frame 12 is rather confusing in that it shows the smaller boy weighing 117.6 pounds and the larger boy weighing 105.9 pounds. Frames 18 and 20 are excellent for showing the difference made by misplaced decimal points. A similar attempt is made in Frames 21–22 but the inference may be a little vague to children of this age. Frame 23 shows Tom weighing himself and he doesn't know whether he weighs 9.98 pounds or 98.7 pounds. This would probably be more effective if the same digits were used. Much of the material might be shown on the blackboard.

Description of FS. 84—Multiplying Decimals (26 frames): The first three frames of this filmstrip show John multiplying common fractions with denominators of ten and one hundred. Next, John multiplies various decimals, and in Frame 10 he writes a rule for locating the decimal point in a product. Following this are a number of problems, including the use of "dollar-and-cents," and automobile mileages. Ten frames are used to demonstrate "rounding off" numbers.

Appraisal: The frequent appearance of the boy John, who is able to work these problems may appeal to people of this age, but most of this material can be shown as effectively on the blackboard.

Description of FS. 85— Dividing Decimals (27 frames): The first twenty-one frames deal with the various types of division involving decimals. John is shown moving the decimal point to its proper position in the quotient. Frames 22–26 show three boys dividing the money they earned by mowing lawns.

Appraisal: The method shown here for placing the decimal point is purely a mechanical method. The material presented here can be shown very well by using the blackboard and play money.

Description of FS. 86—Comparing Decimals (27 frames): The first fifteen frames are used for comparing various decimals by use of pies, money, and weights in a balance. The remainder of the filmstrip shows how batting averages are calculated from the number of "hits" a player gets in a certain number of times at bat.

Appraisal: Here again, the blackboard is just as valuable as a filmstrip in writing numbers to show the class. If a majority in the class are interested in baseball, the section on baseball averages should be useful. The pictures of the baseball players and the listing of data on score cards should be a motivating factor in studying decimals.

Description of FS. 87—Introduction to Percentages (27 frames): Frames 2–4 show situations where percentage is used. The remainder of the filmstrip is given to various examples of percentage—using the percentage of milk in a bottle, percentage of girls in a school, percentage of "red heads" in a class, etc. Percentage is explained as "hundredth of something."

Appraisal: This filmstrip contains a number of pictures that would be more difficult to show otherwise. Some of the examples will be useful for solution in class. Frames 20–23, however, are not clear enough to permit ready understanding of the problem.

Description of FS. 88—Using Percentage (25 Frames): The first seven frames show objects with certain percentages taken away, or remaining. Following this, examples are given of the different types of percentage problems: to find the percent of a number, a number is what percent of another; and find the percent of change between two numbers.

Appraisal: Although a picture accompanies each problem, the pupil must read the problem. There seems to be some doubt in my mind, therefore, whether or not this illustration justifies the price of the filmstrip. There is the possibility of novelty in presenting the problems on the screen.

Description of FS. 89—Problems in Percentage (27 frames): This filmstrip is, as the name implies, a series of problems about percentage.

Appraisal: Most of the pictures seem to add little to the solution of the problems. Frame 19 is probably the best—showing a calendar page representing September. The days that were sunny are represented by small suns, whereas the rainy days are represented by clouds. What percentage of the days were rainy? Projection is an excellent means for showing problems of this type.

General Appraisal of FS. 81 through FS. 89: According to the producer, this series of filmstrips is directed at the intermediate and upper elementary grades. The development of ideas is good and a number of illustrative examples are given. The color is generally good and the pictures are clear. The captions, however, are a little fuzzy, and sometimes difficult to read. (Reviewed by Paul J. Eby, Boston University, General College)

INSTRUMENTS

I. 31—Industrial Drawing Kit (No. 2)

J. L. Hammett Company, Kendall Square Cambridge 42, Mass.

13″×19″; $3.50.

Description: The Industrial Drawing Kit contains a white wooden drawing board, a T-square, and wooden 45- and 30-degree right triangles. The board is cut from a single piece of wood and its dimensions are 5/16″×13″×19″. Wooden cleats attached to the ends by means of countersunk screws prevent the board from warping. The board is of good quality but the T-square and triangles are not transparent. It is often difficult to draw lines correctly when the figure beneath the triangle is hidden. Furthermore, continual rubbing of a pencil against the wooden auxiliary instruments will cause smudges to appear on the drawing paper because carbon adheres to unfinished wood more readily than to celluloid.

When not in use, the drawing board is inverted. There are spaces between the cleats for storing the T-square and triangles.

Appraisal: The kit is quite useful during the study of scales in grades eight and general mathematics courses. It is of use in making accurate geometric and trigonometric diagrams.

The set is sufficiently small to fit on the

student's desk; opaqueness of the auxiliary tools is not a very great disadvantage in the mathematics class, for the drawings required are seldom as complex as those encountered in mechanical drawing courses.

This kit is not designed for portability and is therefore not very appropriate for use during field work despite its convenient size. As a classroom set, however, its price is reasonable; one set for every four or five students is not only a sufficient number but tends to minimize the financial and storage problems. (Reviewed by Bernard Singer, Hyannis, Mass.)

MODELS

M. 19—Circle

Vis-X Company, 1049 South Flower, Los Angeles, Calif.

Disk; 30″ in circumference; ½″ plywood; $2.00.

Description: This circle is a ½″ plywood disk with 30″ circumference, a painted diameter and two movable radii. The disk is encircled by a tape measure permanently attached at one end and snap fastened at the other.

Appraisal: The two movable radii may be placed along the diameter to show that the radius is one-half of the diameter. The tape measure that encircles the disk may be loosened at one end to show the distance that a wheel moves in one revolution is the circumference of the circle. Fastened firmly at one end of the diameter the tape may be folded back and forth along the diameter to show that the circumference is a little more than three times the diameter. It is a simple but valuable concept builder when teaching the circle. (Reviewed by Alice M. Phillipson, Mathematics Consultant, Los Angeles)

M. 20—Linkage

Vis-X Company, 1049 South Flower, Los Angeles, Calif.

Models; 6 steel linkages; $4.00.

Description: Six spring steel linkages, all with a 30″ perimeter, comprise the set of polygons; scalene, isosceles and equilateral triangles, square, rectangle and trapezoid. In a given figure, sides of equal length are the same color, and all may be opened at one vertex.

Appraisal: The linkages each have the same perimeter as the circle to show that the area of regular polygons having the same perimeter increases with the number of sides. These linkages show the rigidity of the three-sided figures and the nonrigidity of the four-sided figures.

The fact that the area of a parallelogram depends upon the base and altitude and not the length of the sides may be shown by changing the linkage from a rectangle to an oblique parallelogram. In each linkage, bars of equal length are painted the same color so that pupils can easily recognize the line value. By opening the linkage so that the sides form a straight line, the perimeter formula is developed. Almost unbreakable, securely fastened at the vertices, easily opened or closed, and occupying little storage space, they are a valuable aid in developing concepts both in mensuration and in geometry. (Reviewed by Alice M. Phillipson, Mathematics Consultant, Los Angeles)

M. 21—Plane Geometry Figures

Vis-X Company, 1049 South Flower, Los Angeles, Calif.

Models; 14 cardboard figures; $2.00.

Description: The set of plane geometric figures is made of cardboard covered with velour paper. They are designed for use on the flocked demonstration board of the Visualizer. The set consists of the following figures all having the same altitude and area; three rectangles 3″×5″, four parallelograms and four trapezoids. Also included are three squares and eighteen square inches. One of each of these groups of figures is cut on the diagonal. The area of a rectangle or square may be developed by the use of square inches. Two of the parallelograms are cut perpendicular to the base so that they may be changed to rectangles to show that the

area is the same. One trapezoid is cut perpendicular to the base through the mid-point of each of the non-parallel sides, so that it may be changed to the original rectangle. The two rectangles with two squares, one 3″ and the other 5″, are designed as a unit to demonstrate $(a+b)^2$. A special color combination is used to make it easy to quickly identify the pieces that belong together. A specially treated demonstration board is required on which to use this set. The company designed a small case for the use of those who do not have a Visualizer available.

Appraisal: These plane figures may be placed on the flocked demonstration board where all can see them. They may be moved by teacher or pupil during the development of the problem. Matching pairs of triangles show the area and congruency of triangles and are valuable in teaching overlapping triangle theorems. Pupils quickly see that some triangles are congruent while others are not. This is a correlated set of basic figures designed to demonstrate visually all of the area formulas. Colorful, attractive, inexpensive, easy to store, it is a valuable addition to the supply of teaching aids of any mathematics teacher. (Reviewed by Alice M. Phillipson, Los Angeles)

M. 22—Cubic Foot

Vis-X Company, 1049 South Flower, Los Angeles, Calif.

Model; 1′×1′×1′; $\frac{1}{4}$″ plywood; $5.00.

Description: This cubic foot is made of $\frac{1}{4}$″ plywood. One side may be removed to permit the inside of the cube to be used for storage. Three sides that have a common vertex are in the plain natural finish. The other three are marked in inch-squares, alternate squares being painted yellow. One view presents the concept of the cubic foot, the other of the cubic inches in the cubic foot.

Appraisal: This cubic foot is well built and the squares are accurately painted. Used with the Visualizer and a set of inch- and half-inch-cubes, the basic concepts of volume may be developed and the ratio of the various units be shown. The removable side permits smaller units such as quarts, gallons, etc., to be placed inside for comparison. (Reviewed by Alice M. Phillipson, Los Angeles)

This section on "Aids to Teaching" has now appeared in the THE MATHEMATICS TEACHER for three years. Your response to the evaluations has been most gratifying. You have given us much help by writing evaluations, by giving us leads to material, and by encouraging us with kind words.

When we began writing this series, we expected to have enough material to keep the section going for about a year. We now find that we are further behind the supply of material than ever before. However, we still need your help. Send us information about material, write reviews on material that you have used in your teaching, let us know about reviews that are not in agreement with your experience. We will be glad to give you credit for your contributions. We want to hear from *you!*

Late in 1950 a letter was sent to all companies whose materials had been reviewed in this section to obtain information as to the continued availability of the material. The results of this survey show that many items are increasing in price—too many to be listed here. The supply of many items has been exhausted because of the demand resulting from reviews appearing in this column. Unfortunately many companies did not reply to the request for information. No survey of films and filmstrips was made since these items are in continuous supply. However, the following materials reviewed in this section are no longer available.

B. 5—Mathematics and The Laws of Nature
B. 8—Navigation
B. 15—Taxes
C. 7—Trigonometric Functions
C. 14—Astronomical Geography Chart
E. 2—Air-Age Research Maps and Charts
E. 28—Add-0
M. 9—Cone, Sphere and Cylinder
P. 6—Portraits of Mathematicians and Pictures of Mathematical Instruments
R. 1—A Tentative Resource Unit on Insurance

The status of the following materials has changed.

B. 22—Computation with Approximate Data. Now available from the Yoder Instrument Co., East Palestine, Ohio
B. 30—An Excursion in Numbers. Orders in excess of one filled at 10 cents a copy.
E. 6—Magic Number Games. Now distributed by Dydo Publishing Co., 1321 Arch Street, Philadelphia, Pa.
E. 26—Calculation. Now distributed by Miss Claire Walker, 260 Oakwood Drive, Wood Dale, Illinois
P. 7—The University Prints. No order filled for less than 25 cents.

NEWS NOTES

A High School Mathematics Conference was held at Millersville State Teachers College, Millersville, Pennsylvania, March 10, 1951. Professor Howard F. Fehr of Teachers College, Columbia University, spoke at the opening morning session on "Implementing a Philosophy of Basic Mathematics." Five demonstration lessons were held at 11:00 A.M. as follows:

I. Basic Mathematics: Grade 9
   Topic: Graphs as Related to Every Day Life
   Teacher: Anna D. Stotz, Columbia High School
   Moderator: Carl Payne, Columbia High School

II. Basic Mathematics: Grades 8–12
   Topic: Mechanical Teaching Aids with Emphasis on Geometry
   Teacher: George R. Anderson, Millersville State Teachers College
   Moderator: Dan W. Witmer, Manheim Borough High School

III. Basic Mathematics: Grades 10–11
   Topic: Visual Aids with Emphasis on Reasoning and Mathematics in Art
   Teacher: Lee E. Boyer, Millersville State Teachers College
   Moderator: V. Anthony Champa, Millersville State Teachers College

IV. Basic Mathematics: Grade 12
   Topic: Writing Checks and Types of Endorsements
   Teacher: Henry U. Walker, Manor-Millersville High School
   Moderator: Rebecca C. Leaman, Manheim Township High School

V. Specialized Mathematics: Grades 10–11
   Topic: Indirect Proof in Geometry
   Teacher: Norman B. Bucher, Lebanon High School
   Moderator: Charles R. Eshleman, Jr., McCaskey High School, Lancaster

Mr. H. W. Charlesworth, President of the National Council of Teachers of Mathematics, spoke at the luncheon meeting and Dr. Catherine A. V. Lyons presented a progress report of the Pennsylvania Council of Teachers of Mathematics at the afternoon session.

The twenty-eighth annual joint meeting of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics, was held at Mississippi State College on February 16 and 17, 1951. Eighty people registered for the meetings of whom twenty-seven were members of the National Council.

Dr. A. S. Householder of the Oak Ridge National Laboratory was the guest speaker at the banquet on the topic "Mathematics in Oak Ridge." A second lecture was on the subject "Nervous Systems and Computing Machines." Additional papers were presented as follows:

"Speak to Me in Calculus." Mrs. W. C. Snipes, Delta State Teachers College

"Mathematics in Music." Mary Ann Turner, Greenwood (Miss.) High School

"General Mathematics for Life Adjustment in the South." Robert W. Lambuth, Jr., Mississippi Southern College

"Motivation in Secondary School Mathematics through History." Robert C. Brown, Southeastern Louisiana College

"The Basic Mathematics Program at Mississippi State." Arthur Ollivier, Mississippi State College

"Social Applications of Eighth Grade Arithmetic." Lucy Douglass, Cleveland (Miss.) High School

"Treatment of Originals in Plane Geometry." Mrs. H. P. Warden, Delhi (La.) High School

"Evaluating a Mathematics Program." Mrs. Margaret Prather, West Tallatchie High School, Webb, Miss.

Newly elected officers for 1951–52 are as follows:

Lester M. Garrison, Louisiana Polytechnic Institute, Chairman

Mary Emma Fancher, Hinds County Agricultural High School, Raymond, Miss., Vice-Chairman

Alva Selman, Holmes Junior College, Goodman, Miss., Secretary

Houston T. Karnes, Louisiana State University, Recorder

Jessie Mae Hoag, Southwestern Louisiana Institute, Louisiana Representative

Virginia Felder, Mississippi Southern College, Mississippi Representative

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Institutes, Workshops, Conferences

(*Continued from page 267*)

enrich and improve the teaching of mathematics (3 weeks, 4½ quarter-hours credit). Write to A. C. VanDusen, Director of the Summer Sessions, Northwestern University, Evanston, Illinois.

University of Wisconsin, Madison, Third Annual Conference on Teaching Mathematics, Grades 1–12, July 2–6, 1951. The program includes addresses and study groups on mathematics and on topics in the teaching of mathematics, curriculum study reports, a mathematics laboratory, and recreation. Housing will be available in university residence halls. If interested, please write to Professor J. R. Mayor, North Hall, Madison 6, Wisconsin for additional information and the complete program.

BOOK SECTION

Edited by JOSEPH STIPANOWICH

Western Illinois State College, Macomb, Illinois

THIS section presents the latest books which have been received for review in THE MATHEMATICS TEACHER. Reviews of many of these books will appear in the monthly issues. Members of the Council are invited to send us further comments or corrections of errors relating to any of the books mentioned. In addition, a free loan service is being arranged whereby any member may borrow any of the books listed for a period not to exceed two weeks. Requests should be addressed to THE MATHEMATICS TEACHER, 212 Lunt Building, Northwestern University, Evanston, Illinois.

BOOKS RECEIVED

ELEMENTARY SCHOOL

1. Supplementary Material

Arithmetic Readiness Cards, Set 2: The Number System, by Maurice L. Hartung, Henry Van Engen, Helen Palmer, 54 picture cards 6½″×8½″, with pictures on both sides in color for the numbers 10–100, and 12-page Teacher's Guidebook. 1950 Scott, Foresman and Company, 433 East Erie Street, Chicago 11, Ill. $3.60.

HIGH SCHOOL

1. General Mathematics

Mathematics, A First Course, by Myron F. Rosskopf, Syracuse University; Harold D. Aten, formerly supervising teacher of mathematics and counselor in the public schools of Oakland, California; and William D. Reeve, Teachers College, Columbia University. Cloth, 472 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. $2.60.

2. Algebra

Self-Help Algebra Workbook, by L. S. Walker, Niles High School, Niles, Michigan; and George E. Hawkins, Lyons Twp. High School and Junior College, La Grange, Illinois. Paper, 80 pages, 1950. Scott, Foresman and Company, 433 East Erie Street, Chicago 11, Ill. $0.80.

COLLEGE

1. Algebra

College Algebra, by Herman K. Fulmer and Walter Reynolds, Georgia Institute of Technology. Cloth, vi+204+xiv pages, 1951. Ginn and Company, Statler Building, Boston 17, Massachusetts. $2.85.

2. Advanced Mathematics

Advanced Mathematics for Technical Students Part II, by H. V. Lowry, Woolwich Polytechnic; and H. A. Hayden, Battersea Polytechnic. Cloth, ix+422 pages, 1951. Longmans, Green and Co., 55 Fifth Avenue, New York. $3.75.

Coefficient Regions for Schlicht Functions, Colloquium Publication, Vol. 35, by A. C. Schaeffer, Purdue University; and D. C. Spencer, Stanford University. Cloth, xi+311 pages, 1950. American Mathematical Society, 531 West 116th Street, New York. $6.00.

MISCELLANEOUS

Better Than Rating, New Approaches to Appraisal of Teaching Services, by the Commission on Teacher Evaluation of the Association for Supervision and Curriculum Development, N.E.A., 1201 Sixteenth Street, N.W., Washington 6, D. C. Paper, 83 pages, 1950. $1.25.

Life Insurance Mathematics, by Robert E. Larson and Erwin A. Gaumnitz, School of Commerce, University of Wisconsin. Cloth, vii+184 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $3.75.

Mathematics, Queen and Servant of Science, E. T. Bell, California Institute of Technology. Cloth, xx+437 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $5.00.

Methods and Materials for Teaching General and Physical Science, by John S. Richardson and G. P. Cahoon, Ohio State University. Cloth, viii+485 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. $4.50.

BOOK REVIEWS

Workbook in Geometry. Virgil S. Mallory and Zaidee T. Horsfall. Chicago, Benjamin H. Sanborn and Company, 1949. Paper, 176 pp., $1.00.

Although this workbook has been prepared to supplement a geometry text, it is organized in a manner that makes it readily adaptable to various teaching methods and materials. This arrangement facilitates quick application to other texts as its eight units are completely subdivided in the table of contents.

Students are provided with an easily available reference, located on the inside of the front cover, containing abbreviations and symbols. Any need for remedial work can be determined with the help of frequent reviews supplemented by a few tests. The continuity of geometry can be emphasized with the use of the large assortment of assumptions, definitions, theorems and corollaries that are enforced by many exercises pertaining to each. The development of meaning is made possible by furnishing numerous applications. The problem of loci, as it appears in both the algebra and geometry courses is considered and proofs for a number of cases are developed.—RODERICK C. MCLENNAN, Arlington Heights Township High School, Arlington Heights, Illinois.

Everyday Arithmetic Junior Books (I and II). Harl R. Douglass, Lucien B. Kinney, and Donald W. Lentz. New York, Henry Holt and Company, 1950. ix+488 pp. and ix+502 pp., $1.88.

These texts, suited to grades seven and eight, may be considered a valuable contribution to the growing list of materials that aim toward better understanding of number and in turn toward the development of skill in dealing effectively with quantitative situations.

Emphasis is put on the functional use of numbers. The content deals with real-life problems about the home, the school, and various activities of the community. The program offers a wide variety of problem-solving experiences, such as, making problems, finishing problems, estimating answers, matching exercises, discarding irrelevant data, and working on special projects.

Provision is made for cumulative review and for practice on new processes to insure retention of skill. Each chapter includes much practice material, inventory tests to point out need for "extra workouts," "shock absorber" tests before each inventory test, problem-solving helps and problem-solving scales. Individual differences are cared for by means of optional exercises and by use of special projects which should provide rich experience and do much to stimulate interest and enthusiasm.

The pupil of average or above average reading skill will have every reason to find the program very interesting, stimulating, challenging and self-instructing. The development of subject matter is easy to follow, the frequent checks on skills and the clear and concise summaries at the end of each chapter should do much to aid the learner to evaluate understandings and to develop skill in dealing effectively with quantitative situations.—Olive G. Wear, Fort Wayne Public Schools, Fort Wayne, Indiana.

Albert Einstein: Philosopher—Scientist. Paul Arthur Schilpp, Editor. Evanston, Illinois, Library of Living Philosophers, 1949. xvi +781 pages. $8.50.

This volume, like the others in this series, is divided into four parts. First there occurs an intellectual autobiography by Einstein, given both in the original German and in the English translation by Schilpp. The second part consists of evaluations of various phases of Einstein's life and work by twenty-five outstanding physicists, mathematicians, and philosophers. This part occupies about two-thirds of the volume. Thirdly there are remarks by Einstein regarding the articles in the second part, and finally there is an exhaustive bibliography of the writings of Albert Einstein by Margaret C. Shields. The book has that characteristic of careful work—a very complete index.

Throughout the book one sees evidence that the modern physicist must be a philosopher at heart, and that the successful philosopher must have an understanding of modern physics. Einstein's autobiography is to a large extent philosophical, although it is primarily a description of his work in developing the special and general theories of relativity. It is definitely not an introduction to these theories, however, and probably cannot be read with understanding by anyone who is not already somewhat familiar with his work.

This same remark applies to most of the articles in the second part. This should not be interpreted to mean, however, that one cannot obtain much interesting information and many new ideas even if one cannot follow the technical aspects of these articles.

For a general, non-technical discussion of Einstein's work the article by Louis de Broglie is to be recommended. Niels Bohr's article consists of discussions which he has had with Einstein concerning problems in epistemology as influenced by developments in atomic physics. It contains descriptions, with illustrations, of physical experiments having bearing on these problems. For Einstein's general philosophy of physics the reviewer recommends the articles of Phillip Frank, Hans Reichenbach, and F. S. C. Northrop.

Those interested in the geometry which has been found useful in Relativity Theory should enjoy the articles by K. Menger and H. P. Robertson. Menger's article is actually a survey of old and new ideas in geometry, and it can hardly fail to be interesting to a mathematician regardless of his degree of interest in the Relativity Theory. Robertson's article is mostly a discussion of the problem of constructing a geometry applicable to the physical world, and it is in this sense that the title "Geometry as a Branch of Physics" must be interpreted.

Any attempt to mention all the articles in the book would swell this review beyond its bounds. The articles mentioned above were those which appealed particularly to the reviewer, and are not meant to be representative in any meaningful sense.

I believe this volume is the best available storehouse of argument for and against the ideas at the forefront of much of today's physics and philosophy. It belongs on the book-shelf of everyone seriously interested in these ideas.—Morton L. Curtis, Northwestern University, Evanston, Illinois.

Instructional Tests in Plane Geometry, Revised Edition. Florence C. Bishop and Manley E. Irwin. Yonkers-on-Hudson, World Book Company, 1950. x+68 pp., $.56.

This booklet consists of forty-five tests which can be used throughout the year with any standard textbook in plane geometry. Although the tests are arranged to follow the general order of topics in geometry textbooks, they may be used in any order to correspond with a particular course of study, text, or manner of presenting a logical and sequential development of the subject based on many fundamental facts and principles.

Each test covers a definite phase of work. Objective-type questions are used in testing the more than one thousand selected items involving exercises and applications related to definitions, theorems, and corollaries. Answers may be written in the booklet. Time for each test ranges from ten to twenty-five minutes.

The coefficient of reliability of most of the tests was about .70. However, the authors prepared the tests to assist in the teaching and the learning of the fundamentals of geometry, to provide a determinant for the amount and kind of remedial work needed. They are not purposed to be a measure of achievement. Validity was obtained "through the judicious selection of criteria to be used in the preparation of each test item, the tryout, and the careful revision."

The booklet includes both graphic and tabular record charts for continuous record of individual and group progress.—E. GLENADINE GIBB, Iowa State Teachers College, Cedar Falls.

Schulreform und Mathematischer Unterricht. Walther Lietzmann. Heidelberg, Quelle und Meyer, 1949. 127 pp., $1.50.

Albeit brief and unpretentious, this is nevertheless a significant and interesting monograph for several reasons: (1) the author is an eminent European educator; (2) he is well-known to many American teachers of mathematics; (3) recent trends in post-war Germany would seem to indicate that many of the problems and needs of mathematical education abroad have become rather similar to ours, and even more interesting to note, are apparently being solved and provided for in much the same way as in the United States.

For example, in discussing the objectives of mathematical education (Chapter 5), Lietzmann points out that it is not the primary purpose of secondary schools to train future mathematicians; that while pupils with mathematical aptitude are not to be neglected, it is incumbent upon the school to meet the cultural needs of less gifted pupils ("usually the non-interested") by getting them to appreciate the significance of mathematics in modern civilization. Furthermore, inductive thinking should be emphasized as well as deductive reasoning. Included among other important objectives are skills in observation; appreciation and understanding of measurement, precision, dimensions, rates, and related ideas; practical applications of mathematics; ethical and esthetic implications of mathematics.

As for methodology (Chapter 6), Lietzmann discusses the advantages and disadvantages of the lecture method, the heuristic method, and the laboratory method; suggesting, among other things, that the lecture method, in modified form, is not to be dismissed as being completely without value.

The next four chapters deal with the selection and organization of curriculum content; with various learning aids, such as textbooks, mathematical instruments, slide rule, drawing instruments, mathematical models, and mathematical exhibits; with examinations, promotions and assigned work. The important principles of curriculum construction are as follows: (1) the psychological (propaedeutic) principle; (2) the integration (fusion) principle; (3) the principle of using practical applications and "real life" situations.

Earlier chapters include a historical sketch, followed by intriguing discussions of the psychology of learning, the factors affecting success in learning, the nature of mathematics, and the education of mathematics teachers. Taken all in all, one is as much impressed with the similarity in mathematical education in the United States and Germany today as with the contrast between them forty years ago. The volume should be on the shelves of the mathematics department of every teachers college and school of education.—WILLIAM L. SCHAAF, Brooklyn College, Brooklyn, New York.

Basic Mathematical Analysis. H. Glenn Ayre. New York, McGraw-Hill Book Company, 1950. xvi+584 pp., $5.00.

Since 1917 when the first freshman college text appeared with the title mathematical analysis, there have been more than a score with the same title. One might well ask why there should be another text in this field. That there is a need for a new vision is evident in the changing concept of collegiate education, and the growth of the junior college, the community college, the technical institutes, and others. This text was written specifically to meet the requirements of these newer type educational institutions.

The usual topics are treated in simple, semi-rigorous developments beginning with number and computation, and continuing through the study of algebraic and trigonometric functions. A chapter on the derivative of functions, ending with applications to maxima and minima problems, is included. The results of this chapter are seldom mentioned in the remainder of the text. The analytic geometry of the straight line and conics is treated in the traditional manner, as well as an introduction to three-dimensional geometry of the plane and quadric surfaces. Polar and parametric representation are included. The exponential function is extended to include complex arguments and hyperbolic functions. Tables of powers and roots, common and natural logarithms, and trigonometric, exponential, and hyperbolic functions are appended.

The presentation is clear and simple. The author has managed to select topics that are pertinent to present or needed for the future study of mathematics and to omit highly difficult or unnecessary phases of the various topics. The illustrations and the applications are lucid and pertinent. The text is clearly written for the student, not the instructor. The format and printing make the book easy to read. The wide range of topics enables the instructor to make a choice to suit his mathematical preferences and to adapt to the time allotted to the course.—HOWARD F. FEHR, Teachers College, Columbia University, New York, N. Y.